Mathematics 172 Homework

Recall that if $P(t)$ is the size of a population in year t then

$$\frac{dP}{dt} = \text{rate of change of } P \text{ with respect to } P \text{ in organisms/year}$$

Thus the per capita growth rate is

$$r = \frac{1}{P}\frac{dP}{dt} \quad \text{(organisms/year)/organisms.}$$

This can be rewritten as (just multiply by P) as

$$\frac{dP}{dt} = rP.$$

When r is constant this has solution

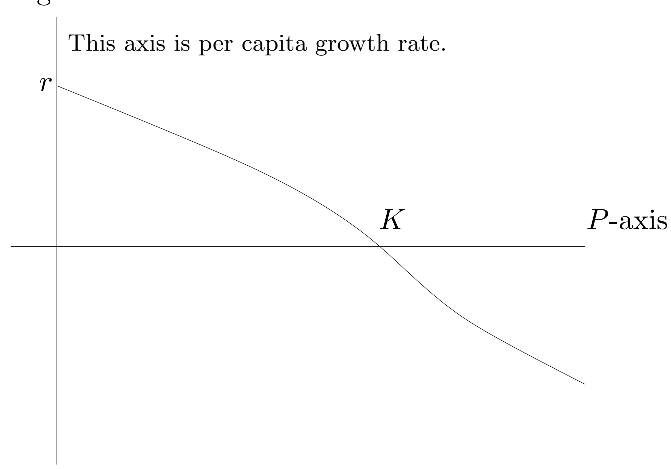
$$P(t) = P(0)e^{rt}.$$

When r is constant and positive, this grows without bound thus is not realistic as a model for modeling the long term behavior of the growth of a population. (But it is still a good model for the modeling the short term growth of a population.)

To get a better model for long term population growth we assume that there is a ***carrying capacity***, K, which is the maximum size population that the environment will support. We then assume that the per capita growth rate r is not constant, but depends on the size of the population. That is

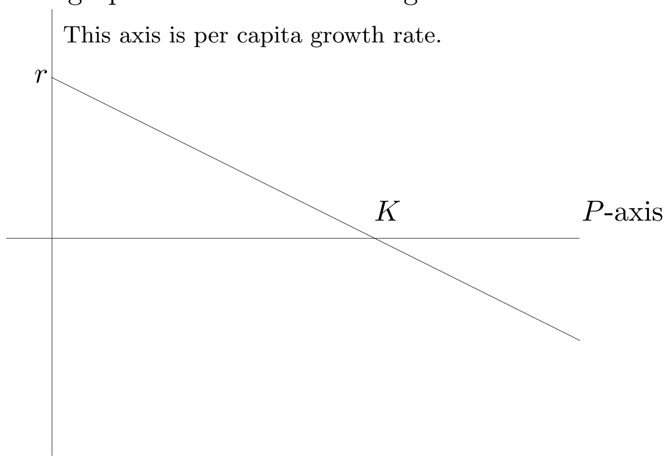
$$\text{per capita growth rate} = f(P) = \text{is a function of the size of the population.}$$

When $P < K$, that is when the size of the population is less than the carrying capacity, the population is growing and thus $r > 0$. When $P > K$, that is the population is larger than the carrying capacity, then the population size is decreasing and thus $r < 0$. This means that the graph if $r = f(P)$ should looking something like



While there are many choices for what the function $f(P)$ could be, the simplest is letting it be a straight line. While in some ways this is an over simplification, the nice thing is that even if we use a more complected curve, the long term predictions are the same.

Here is what the graph looks like for a straight line:



Here r is the instrinstic growth rate. We know how to find the equation of a straight line and the result is

$$\frac{1}{P}\frac{dP}{dt} = \text{per capita growth rate} = r\left(1 = \frac{P}{K}\right)$$

and then multiplication by P gives the ***logistic equation***:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right).$$

1. Find the equilibrium points of the logistic equation and classify them as to being stable or unstable.

For the rest of this term you have to have the logistic equation memorized.

2. For the logistic equation with $r = .2$ and $K = 500$ draw the graphs on the same axis of the equilibrium solutions and also the solutions with $P(0) = 400$ and $P(0) = 600$. For the later two solutions estimate $P(100)$.