Mathematics 172 Homework

Assume that we have a population of organisms that have a negative intrinsic growth rate r. Let $N(t)$ be the number of these organisms at time t. Then N satisfies the rate equation

$$\frac{dN}{dt} = rN$$

and as r is negative all solutions of this tend to zero at t gets large. Now assume that this population is being "stocked" at a constant rate of S. That is organisms are being at a constant rate of S organisms/(time period). Now the rate equation becomes

$$\frac{dN}{dt} = rN + S.$$

Here is an example. Assume that a population of algae is being eaten by snails such that that growth rate of the algae is the negative value $r = -.1$, but that more algae is being added at a rate of $S = 200$ (grams/week). Then the rate equation is

$$\frac{dN}{dt} = -.1N + 200.$$

This has a stable equilibrium point at $N = 200/.1 = 2{,}000$. Thus, in the long run, there will be a stable population size of 2,000 grams for the algae.

1. A tank is being used to raise tilapia. They are being feed duckweed. The fish eat the duckweed so that its intrinsic growth rate is $r = -.3$ (kg/day)/kg. If the tank is stock at a constant rate of 6 kg/day, then what is the stable size of the duckweed in the tank? *Solution:* It is $6/.3 = 20.0$ kg of duckweed.

2. Algae in a tank has an intrinsic of $r = -.05$ (grams/day)/gram. If we wish to have a stable population of algae of size 500 grams, then at what rate should the tank be stocked. *Solution:* Let $A(t)$ be the number of grams of algae in the tank and let S be the stocking rate. Then the rate equation for A is

$$\frac{dA}{dt} = -.05A + S.$$

The equilibrium is $S/(.05) = 20S$. This is the stable population size. We want this to be 500. So set $20S = 500$ and solve for S to get $S = 25$.