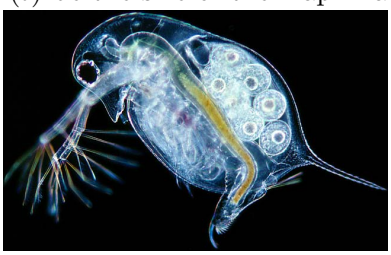
Mathematics 172 Homework

We now look at some modeling problems based on the logistic equation. A population that grows logistically with intrinsic growth rate r and carrying capacity K satisfies

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where $P = P(t)$ is the population size at time t.

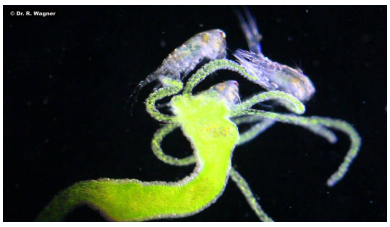
1. A population of Daphnia (aka water fleas) in a bucket grows logistically with intrinsic growth rate $r = .15$ (fleas/day)/flea and carrying capacity $K = 2{,}500$ fleas. Let $P(t)$ be the size of the Daphnia population on day t.



Daphnia

(a) What is the rate equation satisfied by P? *Solution:* $P' = .15P(1 - P/2{,}5000)$.

2. At some point hydras are introduced into the bucket and start to eat the Daphnia 5% of the Daphnia population each day. What is the new rate equation?



Green Hydra eating Daphnia

Solution: As 5% of the population is $.05P$ so the new are equation is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{2{,}500}\right) - .05P.$$

(a) What is the new stable population size of the Daphnia? *Solution:* Solve $.15P\left(1 - \frac{P}{2{,}500}\right) - .05P = 0$ to get that the equilibrium points are $P = 0$ and $P = 1666.67$. The second of these is stable and so this is the new stable population size.

(b) Now suppose that a more aggressive type of hydra is introduced to the bucket and that they eat 20% of the Daphnia per day. What happens his time? *Solution:* The rate equation this time is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{2{,}500}\right) - .2P.$$

and the equilibrium points are $P = 0$ and $P = -833.33$. We can throw out the negative one as population sizes are positive. Now check that 0 is stable. So this time the stable population size is 0, that is the Daphnia population dies out.

3. A population of mosquito fish in a pond grows logistically with $r = .3$ (fish/week)/fish and carrying capacity of $K = 900$.

(a) Bluegill are introduced to the pond and they eat 15% of the mosquito fish each week. What happens? *Solution:* The new stable population size for the mosquito fist is 450.

(b) The bluegill are replaced by large mouth bass, which eat 35% of the population of mosquito fish each week. What happens this time? *Solution:* This time the equilibrium points of the new rate equation are $N = 0$ and $N = -150$ and so the population of mosquito fish dies out.