Mathematics 172 Homework

The solution for these problems are after the last problem.

1. For the system of differential equations

$$\frac{dx}{dt} = .4x \left(\frac{100 - x - .4y}{100}\right)$$
$$\frac{dy}{dt} = .6x \left(\frac{200 - x - .8y}{200}\right)$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the x-y plane) showing

- (a) The lines where $\frac{dx}{dt} = 0$,
- (b) The lines where $\frac{dy}{dt} = 0$,
- (c) The coordinates of all the equilibrium points in the first quadrant. Also say which of the equilibrium points are stable and if which of competitive coexistence, competitive exclusion, x-species dominants, or y-species dominants holds.
- 2. For the system of differential equations

$$\frac{dx}{dt} = .23x \left(\frac{100 - x - 1.5y}{100}\right)$$
$$\frac{dy}{dt} = .07x \left(\frac{150 - x - 4y}{150}\right)$$

draw the phase plane showing

- (a) The lines where $\frac{dx}{dt} = 0$,
- (b) The lines where $\frac{dy}{dt} = 0$,
- (c) The coordinates of all the equilibrium points in the first quadrant. Also say which of the equilibrium points are stable and if which of competitive coexistence, competitive exclusion, x-species dominants, or y-species dominants holds.
- **3.** For the system of differential equations

$$\frac{dx}{dt} = .15x \left(\frac{300 - x - .7y}{300} \right)$$
$$\frac{dy}{dt} = .2x \left(\frac{250 - x - 4y}{250} \right)$$

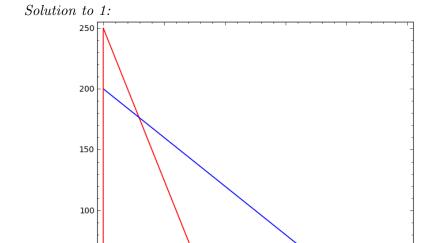
draw the phase plane showing

- (a) The lines where $\frac{dx}{dt} = 0$,
- (b) The lines where $\frac{dy}{dt} = 0$,
- (c) The coordinates of all the equilibrium points in the first quadrant. Also say which of the equilibrium points are stable and if which of competitive coexistence, competitive exclusion, x-species dominants, or y-species dominants holds.
- **4.** For the system of differential equations

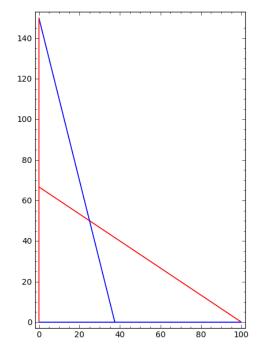
$$\frac{dx}{dt} = .14x \left(\frac{100 - x - 3y}{100}\right)$$
$$\frac{dy}{dt} = .3x \left(\frac{80 - x - .4y}{80}\right)$$

draw the phase plane showing

- (a) The lines where $\frac{dx}{dt} = 0$,
- (b) The lines where $\frac{dy}{dt} = 0$,
- (c) The coordinates of all the equilibrium points in the first quadrant. Also say which of the equilibrium points are stable and if which of competitive coexistence, competitive exclusion, x-species dominants, or y-species dominants holds.

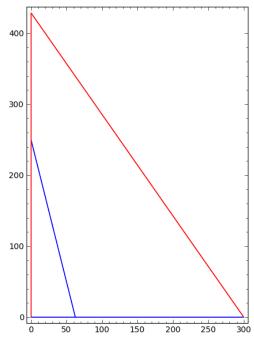


(a) The dx/dt=0 lines are in red. (b) The dy/dx=0 lines are in blue. The equilibrium points are $(0,0),\,(100,0),\,(0,200),\,$ and (29.41,176.5). The point (29.41) is stable and this is competitive coexistence. Solution to 2:



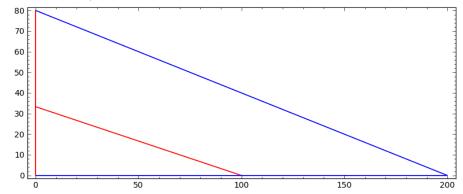
(a) The dx/dt=0 lines are in red. (b) The dy/dx=0 lines are in blue. The equilibrium points are (0,0), (100,0), (0,150), and (25.00,50.00). The points (100,0) and (0,150) are stable. This is competitive exclusion.

Solution to 3:



(a) The dx/dt=0 lines are in red. (b) The dy/dx=0 lines are in blue. The equilibrium points are (0,0), (300,0), and (0,250). The equilibrium points (300,0) is stable. Here the x-species dominants.

Solution to 4:



(a) The dx/dt=0 lines are in red. (b) The dy/dx=0 lines are in blue. The equilibrium points are (0,0),(100,0), and (0,80). The point (0,80) is stable. Here the y-species dominants.