Mathematics 172 Homework

1. A material has a crushing pressure of 32 psi (psi = lb/in^2). A cube with edge of 8 inches made from this material weighs 40 lbs. What is the critical size of a cube made from this material before it is crushed under its own weight? *Solution:* If the cube is scaled (magnified) by a factor of λ, then

$$\text{New weight} = 40\lambda^3.$$

The scaled cube will have a side of length 8λ so

$$\text{Area of base} = (8\lambda)^2 = 64\lambda^2.$$

Therefore the average weight on the base is

$$\text{Average pressure} = \frac{\text{Weight}}{\text{Area of base}} = \frac{40\lambda^3}{64\lambda^2} = .625\lambda \,\text{psi}.$$

Therefore the critical size occurs when

$$.625\lambda = 32.$$

Solving for λ gives

$$\lambda = \frac{32}{.625} = 51.2$$

If the magnification factor is $\lambda = 51.2$, then the edge of the cube of critical size is

$$\text{Critical edge length} = (51.2)(8) = 409.6 \,\text{inches}.$$

□

2. The crushing pressure of red ceder is 4,560 psi. Assume that a red ceder with a height of 5 feet (= 60 inches) has a diameter at the base of 4 inches and weighs 60 lbs. Then what is the critical height where a red ceder crushes itself under its own weight? *Solution:* We magnify by a factor of λ. Then

$$\begin{aligned}
\text{magnified weight} &= 60\lambda^3 \\
\text{magnified base diameter} &= 4\lambda \\
\text{magnified base radius} &= 2\lambda \\
\text{magnified base area} &= \pi(2\lambda)^2 = 12.566\lambda^2.
\end{aligned}$$

$$\text{Averege pressure on the base} = \frac{\text{weight}}{\text{area base}} = \frac{60\lambda^3}{12.566\lambda^2} = 4.775\lambda$$

There the critical size is when

$$4.775\lambda = 4{,}560.$$

Thus the scaling factor at the critical size is

$$\lambda = \frac{4{,}560}{4.775} = 954.97$$

and therefore

$$\text{Critical height} = 954.97(60) = 57{,}298.2 \,\text{in} = 4{,}774.85 \,\text{feet}.$$