Mathematics 172

Quiz 11 Name: Key

You must show your work to get full credit.

The size of a population of fish in a small pond grows by the rule

$$P_{t+1} = P_t + 2.5P_t\left(\frac{K - P_t}{K}\right)$$

where K is a positive constant.

1. Find the equilibrium points of this system. For this problem this will involve using algebra rather than your calculator.

We need to solve The equilibrium points are: 0, K

$P = P + 2.5P\left(\frac{K-P}{K}\right) = f(P)$

$0 = 2.5P\left(\frac{K-P}{K}\right)$

so $P = 0$ or $\frac{K-P}{K} = 0$

i.e the solutions are $P = 0, K$

2. Which of the equilibrium points are stable. (Here you want to use the criterion for stability in terms of the derivative.)

The stable equilibrium points are: There are none

$f(P) = P + 2.5\left(\frac{PK-P}{K}\right) = P + 2.5P - \frac{2.5P^2}{K}$

$f'(P) = 1 + 2.5 - \frac{(2.5)(2P)}{K}$

so $f'(0) = 1 + 2.5 + 0 = 3.5 > 1$. So 0 is unstable.

and $f'(K) = 1 + 2.5 - \frac{2.5(2K)}{K} = 1 + 2.5 - 2(2.5)$

$= -1.5$

and $|-1.5| = 1.5 > 1$. so unstable

3. Do you expect the population size to settle down to a constant or near constant size for this system? Give a yes or no answer and write a sentence or two explaining your answer.

It will not settle down as there is no stable point for it to approach.