Mathematics 172

Quiz 14 Name: Key

You must show your work to get full credit.

1. Let r be a constant and let $P(t)$ satisfy

$$P' = rP, \qquad P(0) = 50, \qquad \text{and} \qquad P(5) = 70$$

Find r. $r =$ .0673

$P' = rP$

implies $P(t) = P_0 e^{rt}$

In our case $P_0 = 50$

so $P(t) = 50e^{rt}$

But $P(5) = 70$ so

$P(5) = 50e^{5r} = 70$ → $e^{5r} = 70/50$

$5r = \ln(70/50)$

$r = \ln(70/50)/5$

$= .0673$

2. Let $N(t)$ satisfy

$$\frac{dN}{dt} = .3N(20 - N)$$

(a) If $N(0) = 5$ what is $N'(0)$? $N'(0) =$ 22.5

$N'(0) = .3N(0)(20 - N(0))$

$= .3(5)(20-5)$

$= 22.5$

(b) If $N(23) = 30$, what is $N'(23)$? $N'(23) =$ −90

$N'(23) = .3(20)(20-30)$

-90

(c) If $N(19) = 20$, what is $N'(19)$? $N'(19) =$ 0

$N'(19) = .3(20)(20-20) = 0$