Mathematics 172

Quiz 18 Name: Key

You must show your work to get full credit.

As was said in class today, a rate equation, also called a differential equation, is an equation (so it has an equal sign in it) and it has a derivative in it. So

$$\frac{dP}{dt} = 5 - 3P$$

is a differential equation as it has both an equal sign and an derivative, but

$$P = 5 - 3P$$

is not because it does not have a derivative in it. The logistic equation is a rate equation so it must have both an equal sign and a derivative in it for it to be correct.

1. Let $N = N(t)$ be the size of a population of rats on an island. Assume that it grows logistically with an intrinsic of $r = .05$ and carrying capacity of $K = 5{,}000$. Write the rate equation for N.

The rate equation is: $\frac{dN}{dt} = .05N(1 - \frac{N}{5000})$

or $N' = .05N(1 - N/5000)$

2. A small pond has a population of mosquito fish that is eaten by a population of small mouth bass in the same pond. Because of the bass the intrinsic growth rate of the mosquito fish is $r = -.1$(fish/month)/fish. (This is not logistic growth, just simple unconstrained decay.) Let $P(t)$ be the size of the mosquito fish population after t weeks.

(a) What is the rate equation satisfied by P? Equation is: $\frac{dP}{dt} = -.1P$

or $P' = -.1P$

(b) To keep the mosquito fish from dying out, the pond is stocked at a continuous rate of $S = 30$fish/month. What is the new rate equation satisfied by $P(t)$?

Equation is: $\frac{dP}{dt} = -.1P + 30$

or $P' = -.1P + 30$

(c) At this stocking rate what is the stable population size of the population of mosquito fish?

Find the equm. pt.

$-.1P + 30 = 0$

$-.1P = -30$

$P = \frac{30}{.1} = 300$

Stable population size is: 300 fish.

(d) If it is desired to have a stable population of 2,000 mosquito fish, then at what rate, S, should the pond be stocked. The rate equation is

$\frac{dP}{dt} = -.1P + S$. The equm pt is when

$-.1P + S = 0$

$-.1P = -S$

$P = 10S$ → we want $10S = 2000$, so

$S = \frac{2000}{10} = 200$

$S =$ 200 fish/month.