Mathematics 172

Quiz 20 Name: Key

You must show your work to get full credit.

Here we will revisit the discrete logistic equation and combine it with the ideas we have just covered. Recall that for a population with per capita growth rate r and carrying capacity K has size N_t in year t, then satisfies the discrete dynamical system

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right).$$

Assume that we have a population of duckweed in a tank and N_t is the number of plants in the tank on day t. Assume this population has discrete logistic growth with $r = .8$ plants/plant and $K = 500$ plants.

1. Write the discrete dynamical system for N_t. $N_{t+1} =$ $N_t + .8N_t\left(1 - \frac{N_t}{500}\right)$

Now assume that a pair of goldfish are added to the tank and they eat 20% of the duckweed each day.

2. Write the new discrete dynamical for N_t. $N_{t+1} =$ $N_t + .8N_t\left(1 - \frac{N_t}{500}\right) - .2N_t$

(substract $.2N_t$ as this is taking out 20%/day)

3. What is the new stable size of the duckweed population?

Stable population size is 375

Find the equilibrium points by solving

$$\cancel{N} = \cancel{N} + .8N\left(1 - \frac{N}{500}\right) - .2N$$

$$0 = .8N\left(1 - \frac{N}{500}\right) - .2N$$

$$= N\left(.8\left(1 - \frac{N}{500}\right) - .2\right)$$

so $N = 0$ or $.8\left(1 - \frac{N}{500}\right) - .2 = 0$

$$.8 - .8\left(\frac{N}{500}\right) = .2$$

$$-.8\left(\frac{N}{500}\right) = -.6$$

and 375 is stable $\quad N = \frac{.6}{.8}(500) = 375$