Mathematics 172

Quiz 25 **Name:**_____

You must show your work to get full credit.

In this quiz we see how to compute the stable age distribution exactly. Let us use the Leslie matrix from the last quiz, that is

$$L = \begin{bmatrix} 0.0 & 2.4 & 16.0 \\ 0.1 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \end{bmatrix}$$

Let

$$\vec{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

be a vector that has the stable age distribution. Then there is a number λ with $L\vec{n} = \lambda\vec{n}$. This becomes

$$\begin{bmatrix} 2.4n_1 + 16n_2 \\ .1n_1 \\ .5n_2 \end{bmatrix} = \begin{bmatrix} \lambda n_1 \\ \lambda n_2 \\ \lambda n_3 \end{bmatrix}$$

which is equivalent to the three equations

(1) $$2.4n_2 + 16n_3 = \lambda n_1$$

(2) $$.1n_1 = \lambda n_2$$

(3) $$.5n_2 = \lambda n_3.$$

1. Use equation (2) to show $n_2 = \dfrac{.1n_1}{\lambda}$. *Solution:* Just divide both sides of (2) by λ.

2. Use the last problem and equation (3) to show $n_3 = \dfrac{.5n_1}{\lambda} = \dfrac{(.5)(.1)n_1}{\lambda^2}$. *Solution:* First divide (3) by λ to get $n_3 = \dfrac{.5n_1}{\lambda}$. Now use the formula for n_2 from the last problem to get

$$n_3 = \frac{.5n_1}{\lambda} = \frac{.5\frac{.1n_1}{\lambda}}{\lambda} = \frac{(.5)(.1)n_1}{\lambda^2}.$$

3. Use the last two problems and equation (1) to show

$$\frac{2.4(.1)}{\lambda^2} + \frac{16(.5)(.1)}{\lambda^3} = 1.$$

Solution: First just plug in the formulas we have for n_2 and n_3 into (1) to get

$$2.4\frac{.1n_1}{\lambda} + 16\frac{(.5)(.1)n_1}{\lambda^2} = \lambda n_1.$$

Each term contains n_1 so we can divide this term out to get

$$2.4\frac{.1}{\lambda} + 16\frac{(.5)(.1)}{\lambda^2} = \lambda.$$

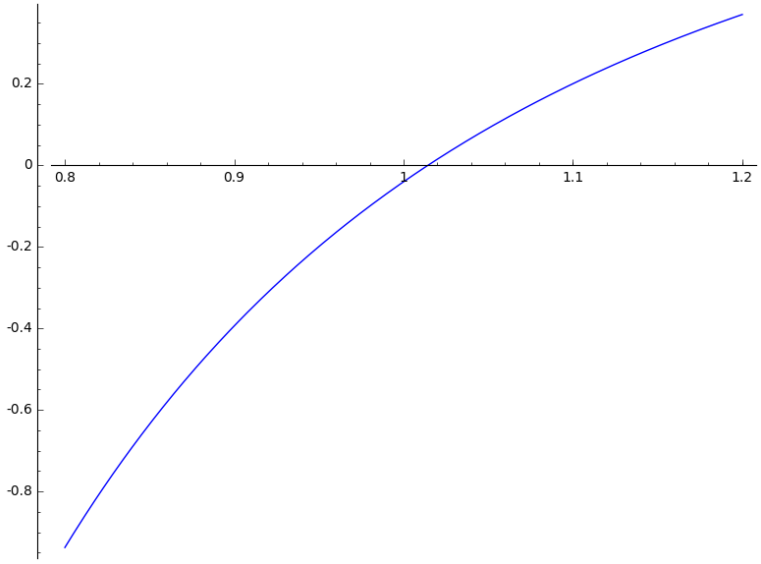
Now dividing by λ gives the desired equation. For use on the next problem note this can be simplified to

$$\frac{.24}{\lambda^2} + \frac{.8}{\lambda^3} = 1.$$

4. Use your calculator to solve this for λ.
Solution: In your calculator let

$\backslash Y_1 =$ 1 -.24/X^2 -.8/X^/3

Use Xmin = .8 and Xmax = 1.2 and then do a ZoomFit to get a graph that looks like



Then do 2nd CALC zero to find that

$$\lambda = 1.0142703$$

5. Now find n_2 and n_3 in terms of n_1. *Solution:* Using our formulas for n_2 and n_3 we find

$$n_2 = \frac{.1n_1}{\lambda} = .0986n_1$$
$$n_3 = \frac{(.1)(.5)n_1}{\lambda^2} = .0486n_1$$

6. Finally give the stable age distribution.
Solution: The total in all the classes is

$$n = n_1 + n_2 + n_3 = n_1 + .0986n_1 + .0486n_1 = 1.1472n_1$$

Thus we have

$$\text{Proportion in Stage 1} = \frac{n_1}{n} = \frac{n_1}{1.1472n_1} = .872 = 87.2\%$$
$$\text{Proportion in Stage 2} = \frac{n_1}{n} = \frac{.0986n_1}{1.1472n_1} = .086 = 8.6\%$$
$$\text{Proportion in Stage 3} = \frac{n_1}{n} = \frac{.0486n_1}{1.1472n_1} = .042 = 4.2\%$$