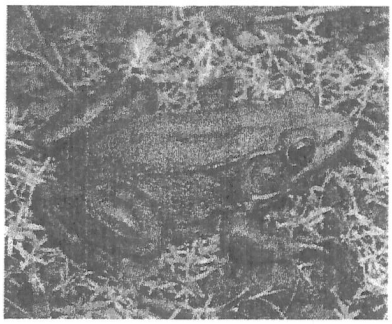
Mathematics 172

Quiz 26 Name: Key

You must show your work to get full credit.



I put a small pound in my backyard two years ago. Last year two adult bronze frogs (*Rana clamitans*) moved in. The resulting population has three stages, tadpoles, juveniles, and adults and these are related by the following Leslie matrix.

$$L = \begin{bmatrix} 0 & 4.8 & 15 \\ .1 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$$

We have seen that if the Leslie matrix is

$$L = \begin{bmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{bmatrix}$$

then the finite growth ratio is the positive root of

$$1 = \frac{f_1}{\lambda} + \frac{p_1 f_2}{\lambda^2} + \frac{p_1 p_2 f_3}{\lambda^2}$$

and if $n_1 = 1$, $n_2 = p_1/\lambda$, $n_2 = p_1 p_2/\lambda^2$ and $n = n_1 + n_2 + n_3$, then stable age distribution is given by the vector

$$\begin{bmatrix} n_1/n \\ n_2/n \\ n_3/n \end{bmatrix}$$

$n_1 = 1$

$n_2 = .1/1.1727 = .0853$

$n_2 = (.1)(.7)/(1.1727)^2 = .0509$

$n = 1.1362$

1. Find λ and the stable age distribution for the frogs in my yard.

Set $Y1 = 1 - (.1)(4.8)/X^2 - (.1)(.7)(15)/X^3$

$X_{min} = .5$ $X_{max} = 1.5$ and 2nd calc zero to find λ

$\lambda =$ 1.1727

Proportion in stage 1 $\frac{n_1}{n} = .880$

Proportion in stage 2 $\frac{n_2}{n} = .075$

Proportion in stage 3 $\frac{n_3}{n} = .0448$

2. Assume that there is no more frogs coming into, or leaving my yard, then starting with the two adults and no tadpoles or juveniles, what is the number and distribution after 20 years.

Number in stage 1 235.61

Number in stage 2 19.27

Number in stage 3 12.34

Proportion in stage 1 $\frac{n_1(20)}{N} = .882$

Proportion in stage 2 $\frac{n_2(20)}{N} = .072$

Proportion in stage 3 $\frac{n_3(20)}{N} = .0462$

Let $[A] = L$ above $\vec{n}(0) = [B] = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$$\vec{n}(20) = L^{20}\,\vec{n}(0) = [A]^{20} * [B] = \begin{bmatrix} 235.61 \\ 19.27 \\ 12.35 \end{bmatrix} = \begin{bmatrix} n_1(20) \\ n_2(20) \\ n_3(20) \end{bmatrix}$$

Total $= n_1(20) + n_2(20) + n_3(20) = 267.32 = N(20)$