Mathematics 172

Quiz 33 Name: Key

You must show your work to get full credit.

A coffee cup left out in a yard fills will rain water and ends up with populations of algae and paramecium growing in it. The paramecium feed on the algae.

$x(t)$ = number of grams of algae in the cup.

$y(t)$ = number of grams of paramecium in the cup.

where time, t, is measured in weeks. We assume that without the paramecium the population of algae grows logistically and that the interaction between the two population is modeled by

$$\frac{dx}{dt} = .8x\left(1 - \frac{x}{100}\right) - 2xy \quad = x\left(.8\left(1-\frac{x}{100}\right) - 2y\right)$$

$$\frac{dt}{dt} = -.1y + .01xy. \quad = y(-.1 + .01x)$$

1. Find the equilibrium points and draw the phase plane showing arrows that indicate the direction of motion.

For $\frac{dx}{dt}=0$ we have

$x=0$ or $.8\left(1-\frac{x}{100}\right)-2y=0$

i.e. $x=0$ or $\frac{.8x}{100}+2y=.8$

x intercept $(100,0)$

y intercept $(0,.4)$

For $\frac{dy}{dt}=0$ we have

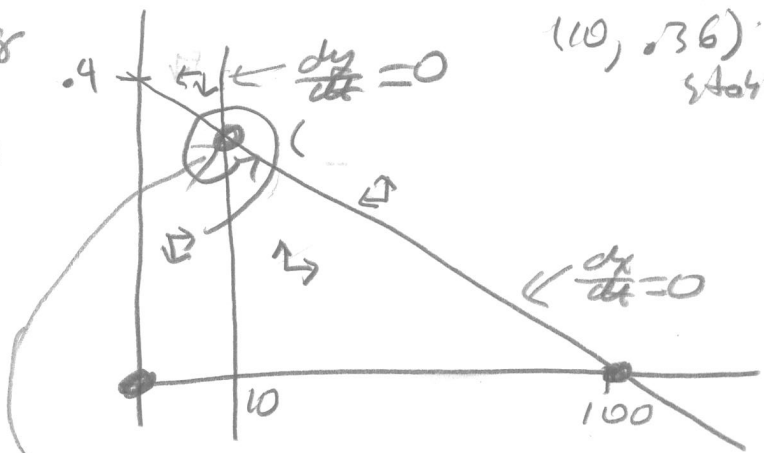
$y=0$ or $-.1+.01x=0$

i.e.

$y=0$ or $x=\frac{.1}{.01}=10$

Equilibrium points: $(0,0)$, $(100,0)$, $(10,0.36)$

$(10, .36)$ is stable



here $x=10$. Use this in $\frac{.8x}{100}+2y=.8$ to get

$.08+2y=.8$

$2y=.72$

$y=\frac{.72}{2}=.36$

2. If $x(0) = 90$ and $y(0) = .5$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 10 $y(100) \approx$ .36

3. If $x(0) = 90$ and $y(0) = 0$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 100 $y(100) \approx$ 0