Mathematics 172 Homework, January 17, 2018.

Let f and g be two functions. Then f and g are ***propositional*** if there is a constant $c > 0$ such that

$$g = cf.$$

That is two quantities are proportional if one is a constant multiple of the other. The constant c is called the ***constant of proportionality***.

A fact that we will use is that if f is proportional to g and g is proportional to h, then f is proportional to h.

Problem 1. If A is proportional to the square of L and $A = 30$ when $L = 5$, then

(a) Find a formula for A in terms of L.

(b) If $L = 20$ what is A

Solution: (a) $A = 1.2L^2$, (b) $A = 480$.

Problem 2. Assume that the cost of wire is proportional to the length of the wire and then 40 feet of wire costs \$4.50, then what is the cost to 220 feet of wire?

Solution: It costs \$24.75.

Problem 3. Assume that the area of a pizza is proportional to its diameter and that the cost is proportional to the area. Also assume that an 10 inch pizza costs 8 dollars.

(a) What is the cost of an 18 inch pizza?

(b) What is the cost of a 24 inch pizza?

(c) If a pizza costs \$100, then what is its diameter?

Solution: (a) \$25.92, (b) \$44.00, (c) The diameter is 35.355 in.

Problem 4. Show that if u is proportional to v and v is doubled, then u doubled. Show that if v is tripled, then u is tripled.

Solution: If u is proportional to v then $u = cv$

$$u = cv$$

for some constant c. Let u_o and v_o be the original values of u and v. Let u_n and v_n be the new values, that is the values after v is doubled. Then, just by what it means to double something, we have

$$v_n = 2v_o.$$

Now using the equation relating u and v get

$$u_n = cv_n = c(2v_o) = 2(cv_o) = 2u_o$$

That is the new value of u is double the original value of u.

In the case that the original value of v is tripled, then the argument is just the same, just replace the 2's in the above with 3's. □

Problem 5. Assume that A is proportional to the square of r, that is for some constant c we have $A = cr^2$. Show that if r is doubled, then the value

of A is multiplied by 4. Show that if r is tripled, then the value of A is multiplied by 9.

Solution: If we denote that A depends on r by $A(r)$, then we have

$$A(r) = cr^2.$$

Doubling r means replacing r by $2r$. The effect on A is

$$A(2r) = c(2r)^2 = c2^2r^2 = c4r^2 = 4cr^2 = 4A(r).$$

That is the value of $A(r)$ is multiplied by 4.

Tripling r is replacing r by $3r$. In this case we have

$$A(3r) = c(3r)^2 = c3^2r^2 = c9r^2 = 9cr^2 = 9A(r)$$

which shows that the value of A is multiplied by 9. □