Mathematics 172 Homework, February 20, 2018.

Problem 1. Let P satisfy

$$\frac{dP}{dt} = 100 - .5P.$$

(a) If $P(9) = 180$ what is $P'(9)$? *Solution:* $P'(9) = 100 - .5(180) = 10$.

(b) If $P(9) = 180$ is P increasing or decreasing at $t = 9$. *Solution:* Since $P'(9) = 10$ and 10 is positive P is increasing at $t = 9$. (Recall that if the derivative is positive that the function is increasing.)

(c) If $P(5) = 230$ is P increasing or decreasing at $t = 5$. *Solution:* The derivative is $P'(5) = 100 - .5(230) = -15$ and this is negative. Therefore P is decreasing at $t = 5$. □

Recall that for an function if x is close to a then we have the approximation

$$f(x) \approx f(a) + f'(a)(x - a).$$

For example if $f(3) = 9$ and $f'(3) = 2$, then we have the approximations

$$f(3.5) \approx f(3) + f'(3)(3.5 - 3) = 9 + 2(3.5 - 3) = 10.0$$

$$f(3.1) \approx f(3) + f'(3)(3.1 - 3) = 9 + 2(3.1 - 3) = 9.2$$

$$f(2.9) \approx f(3) + f'(3)(2.9 - 3) = 9 + 2(2.9 - 3) = 8.8$$

Problem 2. If

$$N' = .1N(2 - N)$$

and $N(5) = 3$ estimate $N(5.5)$, $N(5.2)$, and $N(4.9)$.

Solution: First we compute $N'(5)$.

$$N'(5) = .1(3)(2 - 3) = -.3$$

Now we can approximate as above.

$$N(5.5) \approx N(5) + N'(5)(5.5 - 5) = 3 + (-.3)(5.5 - 5) = 2.85$$

$$N(5.5) \approx N(5) + N'(5)(5.2 - 5) = 3 + (-.3)(5.2 - 5) = 2.94$$

$$N(5.5) \approx N(5) + N'(5)(4.9 - 5) = 3 + (-.3)(4.9 - 5) = 3.03$$