Mathematics 172 Homework, February 26, 2018.

Let us consider small pond where free floating algae is growing. Assume that the pond also has water fleas that eat the algae. Let $A(t)$ be the kg of algae t days after the begging of the year. Assume that the water fleas eat enough that the intrinsic growth rate is

$$r = -.05(\text{grams/gram})/\text{day}.$$

Problem 1. Assuming unconstrained growth, what happens to the algae population in the long run?

Solution: As the growth is unconstrained it satisfies $A' = rA$. That is $A' = -.05A$. This has solution

$$A(t) = A(0)e^{-.05t}$$

and this for large t we have $A(t) \approx 0$. That is the algae dies out of the pond. □

With the same set up assume that there is a flow of water into the pond such that adds algae to the pond at a rate of 50 grams/day.

Problem 2. (a) What is the new rate equation satisfied by A?

(b) What are the equilibrium point(s) of this new equation and which are stable

(c) What is the new stable population size for the algae?

Solution: (a) The new rate equation is

$$\frac{dA}{dt} = -.05A + 50.$$

(b) Solving

$$\frac{dA}{dt} = -.05A + 50 = 0$$

gives that

$$A_* = \frac{50}{.05} = 1{,}000$$

is the only equilibrium point. Sketching a graph show that this is stable.

(c) The new stable population size for the algae is 1,000 grams. □

Problem 3. An agriculturist is raising tilapia. Let $N(t)$ be the number of fish in her tank after t weeks. She keeps harvests them so that the intrinsic growth rate is $r = -.15$ (fish/fish)/week). Thus with no stocking the rate equation for the number of fish is

$$\frac{dN}{dt} = -.15N$$

and so the population would die out. Assume that she wants to have a stable population of 2,000 fish. At what constant rate, S, should she stock (that is add fish to) the tank?

Solution: Letting S be the stocking rate, the rate equation with stocking is then

$$\frac{dN}{dt} = -.15N + S$$

and our goal is to choose S so that the equilibrium population size is 2,000. At the equilibrium size we have $\frac{dN}{dt} = 0$. So plug $\frac{dN}{dt} = 0$ and $N = 2{,}000$ and solve for S. This gives the equation

$$0 = -.15(2{,}000) + S.$$

Therefore

$$S = .15(2{,}000) = 300.$$

Therefore if she stocks at a constant rate of $S = 300$ fish/week the stable population size will be 2,000. □

Problem 4. Assume that a population of duckweed grows logistically in a pond and let $P(t)$ be the number of kg of duckweed in the pond after t days. Assume that the intrinsic growth rate is $r = .2$(kg/day)/kg and that the carrying capacity is $K = 13$kg.

(a) What is the rate equation for the growth of the duckweed?

(b) Assume that at some point some goldfish are added to the pond and that these each the duckweed at a constant rate of 5% of the duckweed/day. What is the new rate equation for $P(t)$/

(c) What is the new stable population of the duckweed after the goldfish are added?

Solution: (a) This is the usual logistic equation:

$$\frac{dP}{dt} = .2P\left(1 - \frac{P}{13}\right).$$

(b) The new rate equation will be

$$\frac{dP}{dt} = .2P\left(1 - \frac{P}{13}\right) - .05P.$$

(c) Doing the usual analysis we find that there are two equilibrium points for the new equation:

$$P_* = 0 \qquad \text{and} \qquad P_* = \left(\frac{.2 - .05}{.2}\right) 13 = 9.75$$

and only $P_* = 9.75$ is stable. So the new stable population size is 9.75. □