Mathematics 172 Homework, February 28, 2018.

Problem 1. Assume that a population of tilapia is being raised in a pool. Assume that they are being harvested at a rate such that the intrinsic growth rate is $r = -.3$ (fish/month)/fish The owner of the pond want to have a stable population of 500 fish. At what rate should he stock the pool to achieve this?

Solution: The rate equation with no stocking is

$$\frac{dP}{dt} = -.3P$$

where $P = P(t)$ is the number of fish the tank after t months. Let S fish/month be the rate the pool is stocked. Then the rate equation with stocking is

$$\frac{dP}{dt} = -.3P + S.$$

We want to choose S so that $P = 500$ is a stable equilibrium point. That is we want

$$0 = -.3(500) + S$$

This gives

$$S = .3(500) = 150 \text{ fish/month}$$

as the desired stocking rate. □

Problem 2. Assume that mosquito fish in a pond have a population that growths logistically with $r = .15$ (fish/week)fish and carrying capacity $K =$ 20,000 fish. Then if $P(t)$ is the population size in week t, then

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{20{,}000}\right).$$

(a) Assume that a fish decease is introduced to the pond that kills off the fish at a rate of 5% of the population per week. Write the new rate equation for P and use it to predict what will happen to the stable size of the mosquito fish population.

(b) Instead of killing off the fish at a rate of 5% assume that they are killed off at a rate of 20% of the population per week. What happens to the mosquito fish population this time?

Solution: (a) The new rate equation is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{20{,}000}\right) - .05P.$$

Either by use of algebra or the calculator we find that the equilibrium points are

$$P = 0 \qquad \text{and} \qquad P = \frac{40{,}000}{3} = 13{,}333.33$$

and graphing a graph shows us that $P = 13{,}333.33$ is stable. So the new carrying capacity is (rounded to the nearest fish) 13,333 fish.

(b) This time the new rate equation is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{20{,}000}\right) - .2P$$

and the equilibrium points are

$$P = 0 \qquad \text{and} \qquad P = \frac{-20{,}000}{3} = -6{,}666.67$$

The second of these can be ignored as a negative population size does not make sense. So the only equilibrium point is $P = 0$ and it can be checked to be stable. Thus the stable population size is $P = 0$. That is in this case the fish population dies out. □

Problem 3. As a variant on the last problem let us again look at the mosquito fish population growing with the logistic equation

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{20{,}000}\right).$$

But this time assume that someone is harvesting the fish at a rate of 500 fish/week to use for mosquito control. What is the new rate equation and what happens to the stable population size of the mosquito fish?

Solution: The new rate equation is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{20{,}000}\right) - 500.$$

Now use your calculator to solve

$$.15P\left(1 - \frac{P}{20{,}000}\right) - 500 = 0$$

to get that the equilibrium points are

$$P = 4226.50 \qquad \text{and} \qquad P = 15{,}773.50$$

Draw a graph to see that the first of these is unstable and the second is stable. Therefore the new stable population size is $P = 15{,}773$ (rounded to the nearest whole fish). □