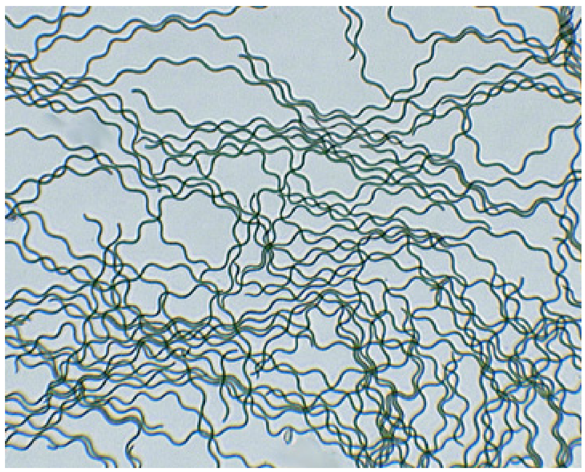
Mathematics 172 Homework, March 2, 2018.

Problem 1. Spirulina (Arthrospira platensis) is a blue-green algae that is raised commercially to sell as a "supper food" for humans and as fish food for fish.



Arthrospira platensis greatly magnified.

An algae farmer grows Arthrospira in a tank where it grows logistically with an intrinsic growth rate of .15 (kg/day)kg and a carrying capacity of 500 kg. This with no interference the tank has a stable population size of 500 kg of algae. What is the maximum daily rate the farmer can harvest the Arthrospira with out kill off the population?

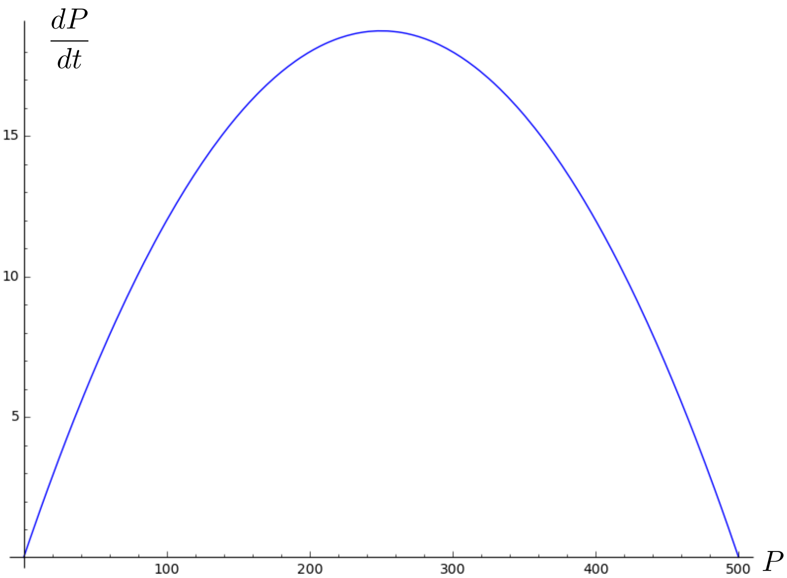
Solution: Before any harvesting, the growth is logistic with rate equation

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{500}\right)$$

where $P = P(t)$ is the number of kilograms of Arthrospira in the tank on day t. We plot dP/dt as a function of P on our calculators by setting

```
\Y1=.15X(1-X/500)
Xmin=0
Xmax=500
```

and then plotting with `ZoomFit` to get a graph that looks something like this:



The horizonal axis is P (the population size) and the vertical axis is dP/dt (the rate of increase of the population in kg/day). Use the calculator to find the maximum rate of change, which is

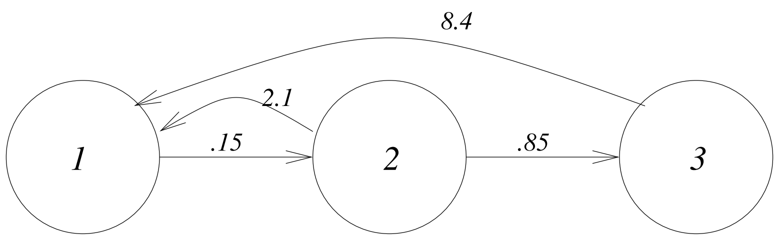
$$\text{Maximum of } \frac{dP}{dt} = 18.75$$

There the maximum rate the farmer can harvest the Arthrospira is 18.75 kg/day. □

Problem 2.

Several types of plant are biennial. That is they live for two years and produce seeds in the second year. Examples are onion, cabbage, parsley, silverbeet, Black-eyed Susan, and carrot. For some of these plant breeders have produced varieties that will produce some small number of seeds in their first year. In the following diagram we have three stages for a type of onion. The first stage is seedlings. The second is juvenile, that is plants that are one year old, and the third stage is adults, plants that are two years old. The plant do not live to a third year.

In this loop diagram the .15 is the proportion of seedlings that survive to be juveniles, the .85 is the proportion of juveniles that survive to be adults, the 2.1 is the average number of seedlings produced by a juvenile and the 8.4 is the average number of seedlings produced by an adult.



If in some year we find there are 98 seedlings, 15 juveniles, and 12 adults, then compute the following:

Number of seedlings in the next year: ________
Number of juveniles in the next year: ________
Number of juveniles in the next year: ________

Number of seedlings in the year after that: ________
Number of juveniles in the year after that: ________
Number of juveniles in the year after that: ________

Solution:

Number of seedlings in the next year: $15(2.1) + 12(8.4) = 132.3$
Number of juveniles in the next year: $98(.15) = 14.7$
Number of juveniles in the next year: $15(.85) = 12.75$

Number of seedlings in the year after that: $14.7(2.1) + 12.75(8.4) = 137.97$
Number of juveniles in the year after that: $132.3(.15) = 19.845$
Number of juveniles in the year after that: $14.7(.85) = 12.495$