Mathematics 172 Homework

The solution for these problems are after the last problem. Recall that an ***equilibrium point*** of the system is a point where both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

1. For the system of differential equations

$$\frac{dx}{dt} = .4x\left(\frac{100 - x - .4y}{100}\right)$$
$$\frac{dy}{dt} = .6y\left(\frac{200 - .8x - y}{200}\right)$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the x-y plane) showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

2. For the system of differential equations

$$\frac{dx}{dt} = .35x\left(\frac{100.0 - x - 1.52y}{100.0}\right)$$
$$\frac{dy}{dt} = .07y\left(\frac{150.0 - 3.75x - y}{150.0}\right)$$

draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

3. For the system of differential equations

$$\frac{dx}{dt} = .33x\left(\frac{300.0 - x - 0.67y}{300.0}\right)$$
$$\frac{dy}{dt} = .51y\left(\frac{250.0 - 4.17x - y}{250.0}\right)$$

draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

4. For the system of differential equations

$$\frac{dx}{dt} = .023x\left(\frac{100.0 - x - 2.86y}{100.0}\right)$$
$$\frac{dy}{dt} = .1y\left(\frac{80.0 - 0.40x - y}{80.0}\right)$$

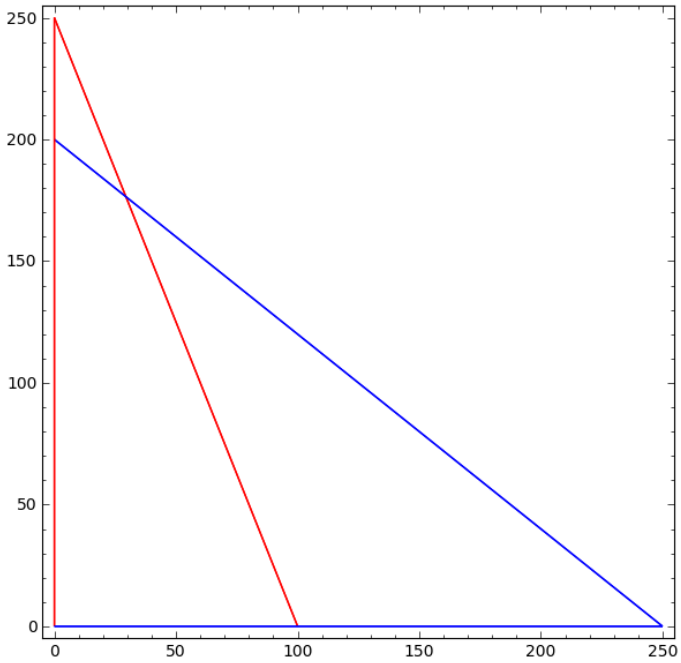
draw the phase plane showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

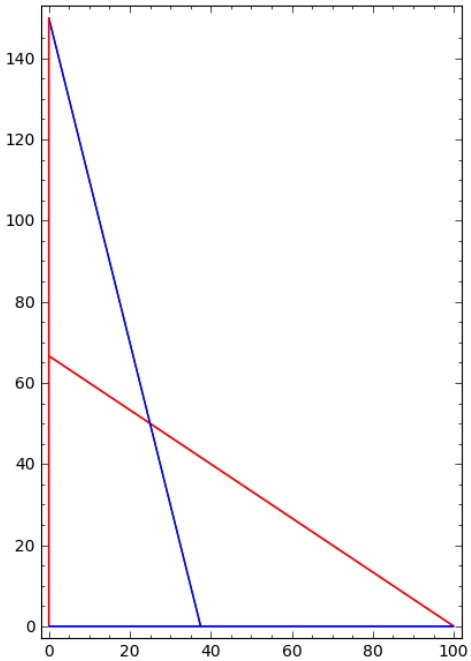
(c) The coordinates of all the equilibrium points in the first quadrant.

Solution to 1:



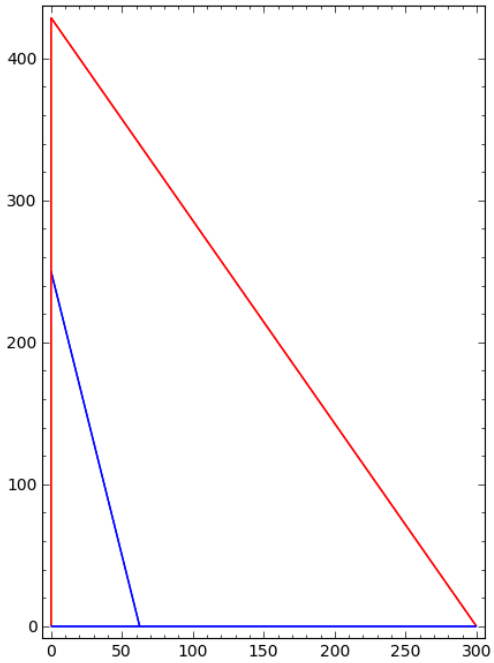
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(100, 0)$, $(0, 200)$, and $(29.41, 176.5)$.

Solution to 2:



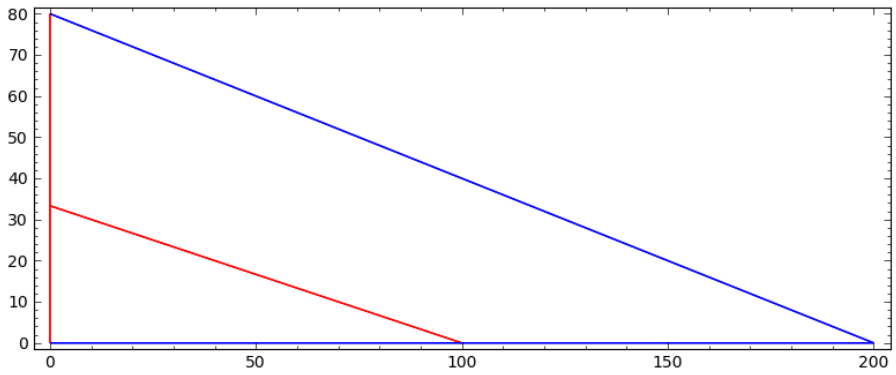
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(100, 0)$, $(0, 150)$, and $(27.2, 47.9)$.

Solution to 3:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(300, 0)$, and $(0, 250)$.

Solution to 4:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$,$(100, 0)$, and $(0, 80)$.