Mathematics 172 Homework

We have seen that for an annual organism that lives and reproduces once a year and has an average number of λ offspring that survive to the next year that if P_t is the population size in year t that

$$P_{t+1} = \lambda P_t$$

where λ is the ***finite growth rate***. This says that to get the size of the population in year $t+1$ we just multiply the population size in year t by λ.

Let

$$P_0 = \text{Size of initial population.}$$

Then we can get the size of the populations for the first several years:

$$\begin{aligned}
P_1 &= \lambda P_0 = P_0\lambda \\
P_2 &= \lambda P_1 = \lambda P_0\lambda = P_0\lambda^2 \\
P_3 &= \lambda P_2 = \lambda P_0\lambda^2 = P_0\lambda^3 \\
P_4 &= \lambda P_3 = \lambda P_0\lambda^3 = P_0\lambda^4 \\
P_5 &= \lambda P_4 = \lambda P_0\lambda^4 = P_0\lambda^5 \\
P_6 &= \lambda P_5 = \lambda P_0\lambda^5 = P_0\lambda^6 \\
P_7 &= \lambda P_6 = \lambda P_0\lambda^6 = P_0\lambda^7
\end{aligned}$$

At this point you see the pattern:

$$P_t = P_0\lambda^t.$$

Example 1. One of the best known examples of an annual insects is the annual cicada which supply so much of the outdoors sound track here is South Carolina in the late summer. Assume that 20 cicadas are introduced on an island and that the finite growth rate for them is $\lambda = 5.5$ cicadas/cicada. Then what is a formula for the number of cicadas after t years? How many cicadas are there in five years? How many long until there are a million rats?

Solution: In this case we have $P_0 = 20$ and $\lambda = 5.5$ Therefore the formula for the number in t years is

$$P_t = P_0\lambda^t = 20(5.5)^t.$$

Therefore after ten years, that is $t = 5$, the number of cicadas is

$$P_{10} = 10(5.5)^5 = 100{,}656.875 \text{ cicadas.}$$

(This can be rounded to the nearest cicada to get $P_5 \approx 100{,}657$ cicadas. Finally to see how long until a million cicadas we want to solve:

$$20(5.5)^t = 1{,}000{,}000 = 10^6,$$

This gives

$$(5.5)^t = (10^6)/20$$

and therefore

$$t \ln(5.5) = \ln\left(10^6/20\right)$$

and thus

$$t = \ln\left(10^6/20\right) / \ln(5.5) = 6.3468487 \text{ years.}$$

1. In a backyard someone introduces 13 annual weeds. Let P_t be the number of weeds t year later. Assume that the finite growth rate of the weeds is 3.3 weeds/weed each year.

(a) Give a formula for the number of weeds after t year.

(b) How long until there are a million weeds in the yard?

(c) How many are there after a 52 years.)

Solution: (a) $P_t = 13(3.3)^t$. (b) Solve $13(3.3)^t = 10^6$ to get $t = 9.423$ year, (c) $P_{52} = 13(3.3)^{52} = 1.1931 \times 10^{28}$ weeds.

Example 2. Assume that 25 sunflowers are introduced into a large field. Sunflowers are annuals. Assume that after three years there are 40 sunflowers in the field. Use this information to find a formula for the number of sunflowers after t years and use this to predict the number that will be in the field after 10 years.

Solution: The number is $P_t = P_0\lambda^t$. We know that $P_0 = 25$, but we still have to find λ. We have

$$P_3 = 25\lambda^3 = 40.$$

This leads to

$$\lambda^3 = (40/25)$$

and thus

$$\lambda = (40/25)^{1/3} = 1.1696\ .$$

Therefore

$$P_t = 25(1.1696)^t$$

Therefore the number after 10 years is

$$P_{10} = 25 * (1.1696)^{10} = 119.77 \text{ sunflowers.}$$

2. Chickweed (Stellaria media) is an annual plant that is considered a weed. Assume that 9 chickweeds are introduced into a large park and that 5 years later there are 100 chickweeds in the park.

(a) Find a formula for the both the number of chickweeds in the park after t years.

(b) How many chickweeds are there in the park after 10 years?

(c) How long until there are 10,000 chickweeds in the park?

Solution: (a) First show that $\lambda = 1.61864$ and therefore $P_t = 9(1.61864)^t$. (b) $P_{10} = 9(1.61864)^{10} = 1{,}111.1$ chickweeds. (c) $t = 9.7812$ years.

Photo of Chickweed.