

## Mathematics 172 Homework, January 29, 2018.

Here are some problems related to what we did today.

**1.** Here is another example related to carbon dating. As said in class the half life of  $^{14}\text{C}$  is 5,730 years.

(a) Give a formula for the percent,  $A(t)$ , of  $^{14}\text{C}$  left after  $t$  years. Give you answer to six decimal places. *Solution:*  $A(t) = 100(.999879)^t$ .

(b) A sample taken from a shroud has only 85.66% of its original  $^{14}\text{C}$  left. How old is it?<sup>1</sup> *Solution:* The age is 1279.13 years.

**2.** The half life of uranium  $^{238}\text{U}$  is  $4.47 \times 10^9$  years. It decays into lead  $^{207}\text{Pb}$ . So by measuring the ratio of  $^{238}\text{U}$  to  $^{207}\text{Pb}$  it is possible to determine the percent of the original  $^{238}\text{U}$  that has decayed.

(a) Give a formula for the percent of  $^{238}\text{U}$  left in a sample after  $t$  billion years. Give you answer to 6 decimal places. *Solution:*  $A(t) = 100(.856358)^t$ .

(b) Rocks have been found in where 59.0239% of the  $^{238}\text{U}$  had decayed into lead. How old are the rocks? *Solution:*  $t = 3.3989$  billion years.

**3.** Rats jump a ship and start to breed on an island. Five years after the rats come to the island a biologist counts the rats on the island and finds that there are 76 of them. She comes back three years after that and finds that there are 93 of them. Assume that the population is growing exponentially. Use this data to estimate the original number of rats that colonized the island, the number  $N(t)$  on the island  $t$  years after they arrived, and how long until the population size becomes 1,000 rats. *Solution:* The formula for the number after  $t$  years is  $N(t) = 54.287(1.0696)^t$ . This gives an estimate of  $N(0) = 54.28$  (which we round off to 54 rats) for the initial population size. Solving  $N(t) = 1,000$  gives that in  $t = 43.30$  years there will be 1,000 rats.

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<sup>1</sup>This data is from 1988 when radiocarbon dating was used find the age of the Shroud off Turin.