Mathematics 172 Homework, January 31, 2018.

What we saw in class today was that if we have a population with a per capita death rate of d organisms/organism and a per capita birth rate of b organisms/organism and we ignore all other influences on the population size, then it N_t is the number of organisms in the population in year t, then

$$\Delta N_t = \text{Change in population size form year } t \text{ to year } t+1 = bN_t - dN_t.$$

Since $\Delta N_t = N_{t+1} - N_t$ the becomes

$$N_{t+1} - N_t = bN_t - dN_t = (b-d)N_t = rN_t$$

where we have set

$$r = b - r = \textbf{\textit{per capita growth rate.}}$$

We can also rewrite this equation as

$$N_{t+1} = N_t + rN_t = (1+r)N_t = \lambda N_t$$

where

$$\lambda = 1 + r = \textbf{\textit{growth factor.}}$$

We have seen that the solution to $N_{t+1} = \lambda N_t$ is

$$N_t = N_0 \lambda^t.$$

Problem 1. 500 bass are released into a lake. Three years later the population size is 812 bass. Use this date to find the per capita growth rate r, a formula for N_t, for the number of bass after 10 years, and how long does it take for the population to reach a size of 10,000.

Solution: Here we are assuming that the growth rate is exponential. Thus

$$N_t = 500\lambda^t.$$

To find the growth factor λ solve

$$N_3 = 500\lambda^3 = 812.$$

This gives

$$\lambda = \left(\frac{812}{500}\right)^{1/3} = 1.1754$$

Therefore the per capita growth rate is

$$r = \lambda - 1 = .1754$$

The formula for N_t is

$$N_t = 500(1.1754)^t.$$

To see when the population reaches 10,000 we solve

$$N_t = 500(1.1754)^t = 10{,}000$$

which gives

$$t = \frac{\ln(10{,}000/500)}{\ln(1.1754)} = 18.536 \text{ years.}$$

□

Problem 2. A population of primates on an island has a per capita growth rate of $r = -.07$ monkeys/monkey. (Note the growth rate is negative and so the population size is decreasing.) If the population size this year is 89 monkeys, then give a formula for the number of monkeys t years from now, the number there will be in ten years, and how long until there are only five monkeys.

Solution: The growth factor is $\lambda = 1 + r = .93$. Thus the number t years from now is

$$N_t = 89(.93)^t.$$

Therefore in ten years the number will be

$$N_{10} = 89(.93)^{01} = 43.07\text{Monkeys}.$$

To see how long until there are only five monkeys left solve

$$N_t = 89(.93)^t = 5$$

which gives

$$t = \frac{\ln(5/89)}{\ln(.93)} = 39.67 \text{ years}.$$