Mathematics 172

Quiz 11 Name: Key

You must show your work to get full credit.

1. Define what it means for two functions f and g to be proportional.

There is a positive constant c such that $f = cg$

2. The largest crocodile ever in captivity was Lolong a saltwater crocodile who lived in an ecotourism park in the Philippines. He was a 6.17 meters long and weighed 1,075kg (That is 20 feet 3 inches and 2,370 pounds.) Lolong is pictured to the right.

Sarcosuchus is a genus of extinct crocodile that lived 112 million years ago in Africa and South America and some of them were over 11 meters long. Assuming that Sarcosuchus had the same proportions as Lolong estimate the weight of a 11 meter specimen.

The magnification factor (=scale factor) is $\lambda = \frac{11}{6.17} = 1.782.$

Weight is proportional to $(\text{length})^3$, so

Weight of sarcosuchus $= (1.782)^3(1{,}075) = 6083.2$ kg

Estimated weight is 6,083.2 kg (=13,411 lbs)

3. A cell has volume $V = 5 \times 10^{-6}\text{mm}^3$ and a surface area of $A = 8 \times 10^{-3}\text{mm}^2$. Assume that CO_2 passes through the cell menbraneat a rate of .4(mg/mm^2)/hr.

(a) What is the total amount of CO_2 coming into the cell per hour?

Amount = (rate) × (Area)

$= .4\,(8 \times 10^{-3})$ mm^2 (mg/mm^2)/hr

$= .0032$ mg/hr

Amount of CO_2/hour is .0032 mg/hr.

(b) If the cell needs 60(mg/mm^3)/hr of CO_2 to survive, then how much can it be magnified before it dies from lack of CO_2? If we magnify by a factor of λ then the area is $A_\lambda = 8 \times 10^{-3}\lambda^2$ mm^2

Magnification factor is 10.667

The total amount of CO_2 is $A_\lambda \times (.4) = .0032\lambda^2$ mg/hr.

The new volume is $V_\lambda = 5 \times 10^{-6}\lambda^3$ mm^3. So

(Amount of CO_2)/Volume $= \frac{.0032\lambda^2}{5 \times 10^{-6}\lambda^3} = \frac{640}{\lambda}$ (mg/mm^3)/hr

The cut of is when $\frac{640}{\lambda} = 60$, ie $\lambda = \frac{640}{60} = 10.667$

4. It is know that some mice were released on an island. Two years after the release there were 100 mice and three years after that there were 140 mice. Let P_t be the size of the mouse population t years after there release, and assume that N_t grows exponentially.

(a) Give a formula for N_t. $N_t = N_0 \lambda^t$ $\qquad N_t =$ $79.86\,(1.119)^t$

$N_2 = N_0 \lambda^2 = 100$

$N_5 = N_0 \lambda^5 = 140$

so

$N_0 = \frac{100}{\lambda^2} = \frac{140}{\lambda^5}$

$\to \lambda^3 = \frac{140}{100}$

$\lambda = (\frac{140}{100})^{\frac{1}{3}}$

$= 1.119$

$N_0 = \frac{100}{\lambda^2} = \frac{100}{(1.119)^2} = 79.86$

(b) What is a good estimate for the original number of mice released?

$N_0 \approx$ $79.86 \approx 80$

(c) What is the per capita growth rate? $r = \lambda - 1$ $\qquad r =$ $.119$

(d) How long until there are 10,000 mice? Time to 10,000 mice. 42.96 years.

Solve $(79.86)(1.119)^t = 10{,}000$

$(1.119)^t = 10000/79.86$

$t \ln(1.119) = \ln(10000/79.86)$

$t = \ln(10000/79.86)/\ln(1.119)$

$= 49.95$ years

5. A population of fish is a pond has size P_t and assume that it satisfies the discrete dynamical system

$$P_{t+1} = \frac{.5P_t^2}{1 + .001P_t^4}$$

Plot this function on your calculator with `Xmin=0` and `Xmax=20`

(a) Assuming that $P_0 = 15$ find the following

$P_1 =$ 2.179

$P_2 =$ 2.322

$P_3 =$ 2.620

(b) Find the equilibrium points and determine which of them are stable and which are unstable.

$P_* = 0$, dy/dx = slope = 0 so stable

$P_* = 2.034$ $\frac{dy}{dx}$ = slope = 2.00 so unstable

$P_* = 7.11$ $\frac{dy}{dx}$ = slope = $-.853$ so stable