Mathematics 172

Quiz 12 Name: key

You must show your work to get full credit.

1. Find the derivatives of the following:

(a) $y = 3e^{2x}$. $y' =$ $6e^{2x}$

(b) $A = 6e^{r^2}$. $\frac{dA}{dr} = 6e^{r^2}(2r) = 12re^{r^2}$ $\frac{dA}{dr} =$ $12re^{r^2}$

(c) $P(t) = 15e^{rt}$ where r is constant. $P'(t) =$ $15re^{rt}$

2. If r is a constant and $P'(t) = rP(t)$ show that

$$P(t) = P(t)e^{rt}$$

Hint: Take the derivative of $f(t) = e^{-rt}P(t)$.

$f'(t) = (e^{-rt})'P(t) + e^{-rt}(P(t))'$

$= -re^{rt}P(t) + e^{-rt}rP'(t)$ (used $P'(t) = rP(t)$)

$= -re^{rt}P(t) + re^{-rt}P(t)$

$= 0$

Thus $f = c =$ a constant.

$f(0) = e^{-0}P(0) = P(0) = c.$

so $c = P(0)$

$f(t) = e^{-rt}P(t) = c = P(0)$

so $P(t) = P(0)e^{rt}$

3. Solve the initial value problem

$$P' = .15P(t), \qquad P(0) = 500.$$

$P(t) =$ $500e^{.15t}$

4. Assume that $P'(t) = P(t)(20 - P(t))$ and that $P(0) = 10$. Then what is $P'(10)$?

$P'(10) = P(10)(20 - P(10))$

$= 10(20 - 10)$

$= 10 \times 10 = 100$

$P'(10) =$ 100