Mathematics 172

Quiz 13 Name: Key

You must show your work to get full credit.

1. Let $P(t)$ satisfy the initial value problem

$$\frac{dP}{dt} = .15P \quad \text{and } P(0) = 750.$$

(a) Give a formula for $P(t)$. $P(t) = 750e^{.15t}$

We know $P' = rP$

Implies $P(t) = P(0)e^{rt}$

(b) What is $P(100)$?

$P(100) = 750e^{.15(100)}$ → $P(100) = 2,451,763,029 \approx 2.451 \times 10^9$

2. Let A satisfy

$$A'(t) = rA(t) \quad \text{and } A(0) = 12.5$$

where r is a constant. Assume that $A(5) = 14.7$.

(a) Find r. $r = .0324$

$A(t) = 12.5e^{rt}$

$A(5) = 12.5e^{5r} = 14.7$

$e^{5r} = \frac{14.7}{12.5}$

$5r = \ln(14.7/12.5)$

$r = \ln(14.7/12.5)/5 = .0324$

(b) What is $A(20)$? $A(20) = 23.944$

$A(20) = 12.5e^{.0325(20)}$

$= 23.944$

3. Assume N satisfies

$$\frac{dN}{dt} = .1N\left(1 - \frac{N}{20}\right)$$

If $N(3) = 25$, what is $N'(3)$?

$N'(3) = .1N(3)\left(1 - \frac{N(3)}{20}\right)$ $N'(3) = -.625$

$= .1(25)\left(1 - \frac{25}{20}\right)$

$= -.625$