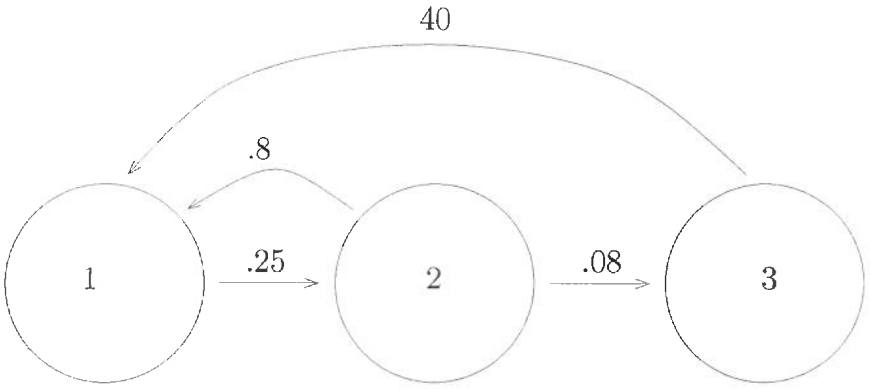
Mathematics 172

Quiz 21 Name: Key

You must show your work to get full credit.

We are studying a biannual plant with three stages. The first is seedling, the second is juveniles and the third is adults. The life history of a population of these plants on a small island is summarized in the following loop diagram:



1. What is the average number of offspring of an adult that survive to be seedlings?

The number is 40

2. What is the proportion of juveniles that live to be adults?

The proportion is .08

3. What is the Leslie matrix.

$L = [A] =$ $\begin{bmatrix} 0 & .8 & 40 \\ .25 & 0 & 0 \\ 0 & .08 & 0 \end{bmatrix}$

We now assume that the island originally had none of the plants, but then we plant 1,000 seedlings.

4. How many plants are there in each stage the next year?

Number in stage 1 0

Number in stage 2 250

Number in stage 3 0

for some number λ. To simplify notation let $n_j = n_j(t)$. Use the loop diagram to derive the equations

$$n_1(t+1) = \lambda n_1 = .8n_2 + 40n_3$$
$$n_2(t+1) = \lambda n_2 = .25n_1$$
$$n_3(t+1) = .\lambda n_3 = .08n_2$$

Use the last two of these to show

$$n_2 = \frac{\lambda}{.08} n_3$$
$$n_1 = \frac{\lambda^2}{(.08)(.25)} n_3$$

and use these in the first equation to get

$$\lambda \frac{\lambda^2}{(.08)(.25)} n_3 = .8 \frac{\lambda}{.08} n_3 + 40n_3$$

which can be rewritten as

$$1 = \frac{.8(.25)}{\lambda^2} + \frac{40(.08)(.25)}{\lambda^3}$$

8. Use your calculator to find the positive solution to this equation.

$\lambda =$ _____

9. Now find n_1 and n_2 in terms of n_3.

$n_1 =$ _____ $n_2 =$ _____

10. Find the stable age distribution.

Proportion in stage 1 _____

Proportion in stage 2 _____

Proportion in stage 3 _____

5. How many plants are there in each stage after 5 years?

Number in stage 1 320

Number in stage 2 10

Number in stage 3 16

6. Compute the proportion in each stage after 50 years and after 51 years.

Number in stage 1 after 50 years 357.4

Number in stage 2 after 50 years 89.80

Number in stage 3 after 50 years 7.08

Number in stage 1 after 51 years 355.2

Number in stage 2 after 51 years 89.35

Number in stage 3 after 51 years 7.18

7. What is the stable age distribution?

Proportion in stage 1 _____

Proportion in stage 2 _____

Proportion in stage 3 _____

Let

$$N(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{bmatrix}$$

where $n_j(t)$ is the number in stage stage j in year t. If we have reached the stable age distribution then

$$N(t+1) = \lambda N(t)$$