Mathematics 172

Quiz 22 Name: Key

You must show your work to get full credit.

1. Solve the following:

$$\frac{dN}{dt} = .15N \quad \text{and} \quad N(0) = 100.$$

$N(t) =$ $100e^{.15t}$

2. Assume that $P(t)$ satisfies

$$\frac{dP}{dt} = .05P\left(1 - \frac{P}{200}\right)$$

(a) If $P(0) = 20$ what is $P'(0)$? $P'(0) =$ $.9$

$.05(20)(1 - \frac{20}{200}) = .9$

(b) If $P(10) = 180$, what is $P'(10)$? $P'(10) =$ $.9$

$.05(180)(1 - \frac{180}{200}) = .9$

(c) If $P(10) = 180$, estimate $P(10.5)$. $P(10.5) \approx$ 180.45

(d) If $P(10) = 180$, estimate $P(100.5)$ $P(105.5) \approx$ 200

carrying capacity

$P(10.5) \approx P(10) + P'(10)(10.5 - 10)$
$= 180 + .9(.5) = 180.45$

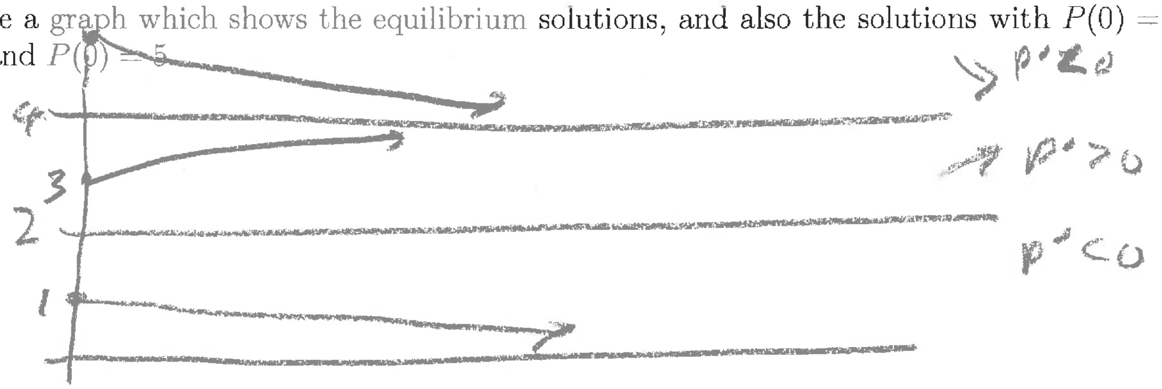
3. Let $P(t)$ satisfy the rate equation

$$\frac{dP}{dt} = .01P(P-2)(4-P).$$

(a) What are the equilibrium points of this equation?

solve $.01P(P-2)(4-P) = 0$ The equilibrium points are $0, 2, 4$

(b) Make a graph which shows the equilibrium solutions, and also the solutions with $P(0) = 1$, $P(0) = 3$ and $P(0) = 5$



(c) If $P(0) = 5$ estimate $P(87)$. $P(87) \approx$ $4.$

4. Let $A(t)$ be the number of grams of algae in a tank after t days. Assume that the algae grows logistically with an intrinsic growth rate of .08 (grams/day)/gram and carrying capacity of 125 grams.

(a) What is the rate equation for A? Equation is $\frac{dA}{dt} = .08A\left(1-\frac{A}{125}\right)$

(b) If $A(0) = .1$ grams, estimate the amount of algae in the tank a half day later.

$$A'(0) = .08(.1)\left(1-\frac{.1}{125}\right)$$
$$= .0079936 \approx .008$$

$A(.5) \approx$ .104

$$A(.5) \approx A(0) + A'(0)\,(.5-0)$$
$$= .1 + (.008)(.5)$$
$$= .104$$

(c) If $A(0) = .1$ estimate the amount of algae in the tank 50 days later.

$A(50) \approx$ 125

Carrying capacity

5. A population of paramecium lives in a cup left out in a yard. The population originally grows logistically with an intrinsic growth rate of $r = .5$ (paramecium/day)/paramecium and a carrying capacity of $K = 500$ paramecium. At some point a population of rotifers, starts in the cup and eat 20% of the paramecium each day. What is the new stable population size for the paramecium population.

Stable population size is 300

Let $P(t)$ = number of paramecium on day t.

The rate equation is then

$$\frac{dP}{dt} = \underbrace{.5P\left(1-\frac{P}{500}\right)}_{\text{logistic part}} - .2P$$

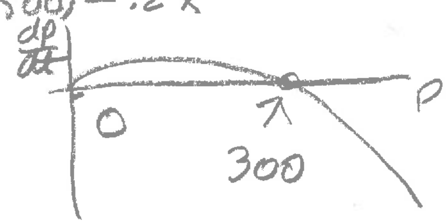
take out 20%

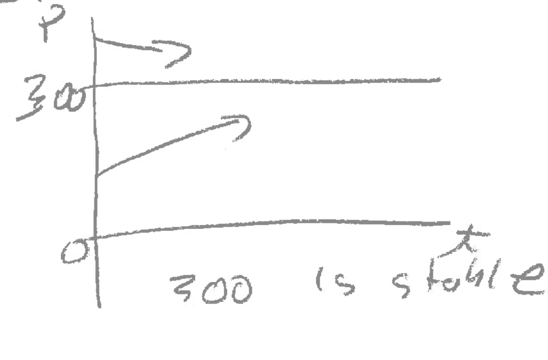
solve $.5P\left(1-\frac{P}{500}\right) - .2P = 0$ using calculator

$Y1 = .5X(1-X/500) - .2X$

Xmin = 0

Xmax = 500





300 is stable

6. A population of fish is being raised for food in a pond. Assume that they are being harvested at a rate such that the intrinsic growth rate of the population is $r = -.05$ (fish/month)/fish. At what rate should the pond be stocked to have a stable population size of 500 fish?

Stocking rate is: 25 fish month.

Let $N(t)$ = number of fish in month t. Let S = stocking rate.
Then

$$\frac{dN}{dt} = -.05N + S.$$

If $N = 500$, then this is equilibrium point and so $\frac{dN}{dt} = 0$. Thus

$$0 = -.05(500) + S$$

$$S = .05(500) = 25$$

7. A population of insects is growing logistically with $r = .3$(bugs/week)/bug and $K = 900$ bugs.

(a) What is the stable population size? Stable size is 900

carrying capacity

So rate equation is

$$\frac{dN}{dt} = .3N\left(1 - \frac{N}{900}\right)$$

(b) If a predator is introduced that eats the insects at a rate of 30 bugs/week what is the new stable population size?

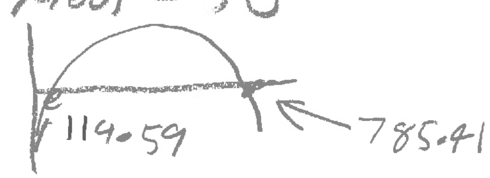
New stable size is 785 bugs.

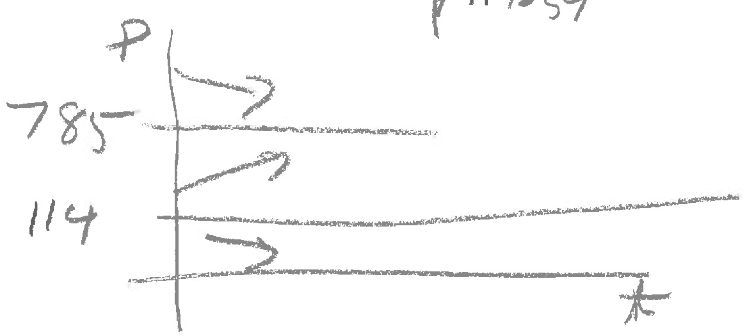
The new rate equation is $\frac{dN}{dt} = .3N\left(1 - \frac{N}{900}\right) - 30$

\Y1 = .3X(1 - X/900) - 30

Xmin = 0

Ymax = 900

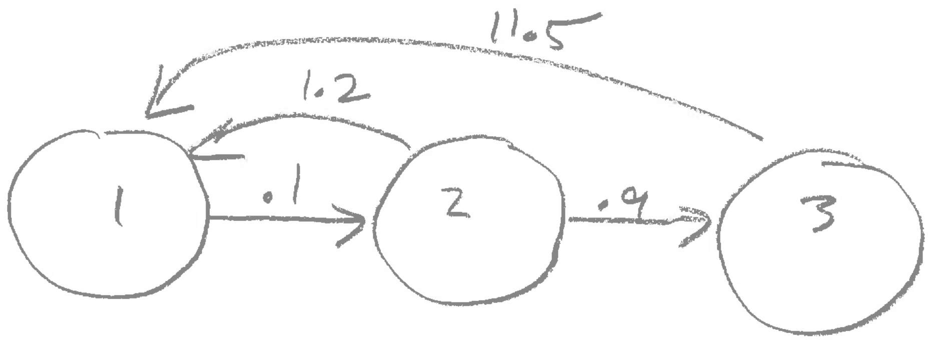




8. A population of weeds in a yard has three stages, seedlings, juvenals, and adults. The Leslie matrix for this population is

$$L = \begin{bmatrix} 0 & 1.2 & 11.5 \\ .1 & 0 & 0 \\ 0 & .9 & 0 \end{bmatrix}$$

(a) Draw the loop diagram.



(b) What does the number 1.2 mean? It is the average number of off spring to a stage 2 org. that live to stage 1.

(c) What does the number .1 mean? It is the proportion of stage 1 orgs. that survive to stage 2

(d) If we start with a population of 98 in stage 1, 20 in stage 2, and 9 in stage that find the following

The number in stage 1 the next year. 127.5

The number in stage 2 the next year. 9.8

The number in stage 3 the next year. 18

The number in stage 1 after 50 years. 1681.6

The number in stage 2 after 50 years. 165.81

The number in stage 3 after 50 years. 135.27