Mathematics 172

Quiz 24 Name: key

You must show your work to get full credit.

Let $x(t)$ and $y(t)$ be the population size of two competing species. Assume the changes in the population sizes are modeled by

$$\frac{dx}{dt} = .1x\left(\frac{100 - x - 2y}{100}\right)$$

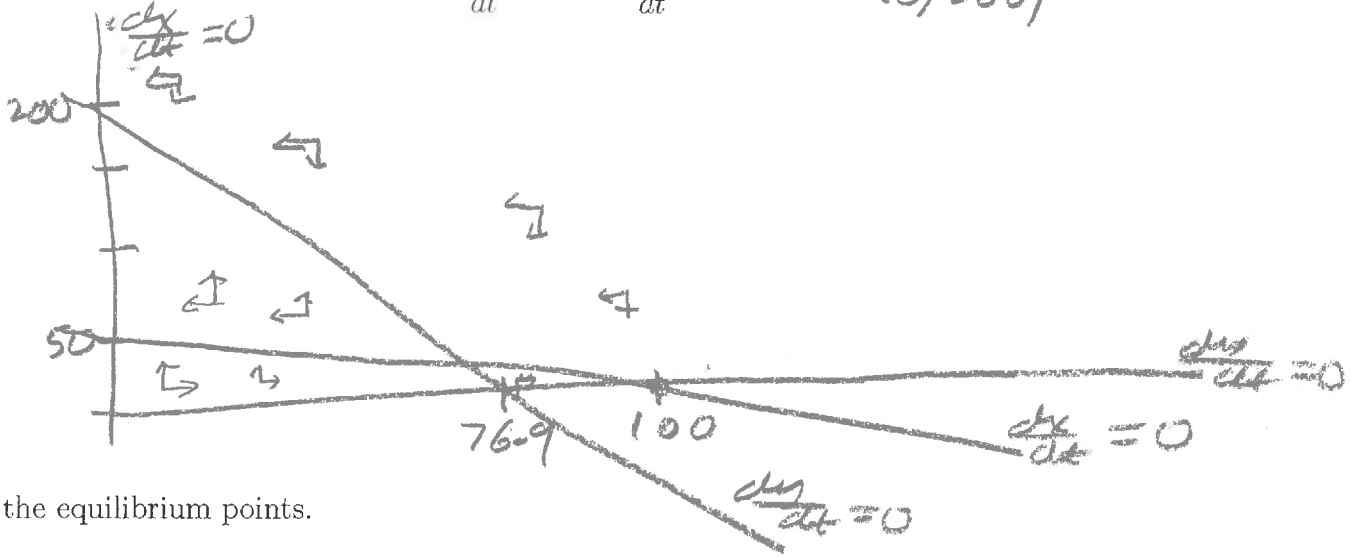
$$\frac{dy}{dt} = .15y\left(\frac{200 - 2.6x - y}{200}\right).$$

$x + 2y = 100$
Intercepts $(100, 0), (0, 50)$

$2.6x + y = 200$
Intercepts $\left(\frac{200}{2.6}, 0\right) = (76.9, 0)$

$(0, 200)$

1. Make a graph showing the lines where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.



2. Find the equilibrium points.

The equilibrium points are: $(0,0), (100,0), (0,200), (71.4, 14.3)$

From the picture we see that $(0,0)$, $(100,0)$, $(0,200)$ are equilibrium pts. To get the last one solve

$x + 2y = 100$
$2.6x + y = 200$ $\times 2$
$5.2x + 2y = 400$

$-4.2x = 300$

$x = \frac{300}{4.2} = 71.4$

so

$y = 200 - 2.6x$
$= 200 - 2.6(71.4)$
$= 14.3$

3. Add arrows to your picture showing the direction that the point $(x(t), y(t))$ is moving in the regions where $\frac{dx}{dt} \neq 0$ and $\frac{dy}{dt} \neq 0$