Mathematics 172

Quiz 29 Name: Key

You must show your work to get full credit.

In a predator-prey system let

$x(t)$ = size of prey population at time t, $y(t)$ = size of predator population at time t.

and assume these satisfy the system of rate equations

$$\frac{dx}{dt} = .1x\left(1 - \frac{x}{500}\right) - 5xy = x\left(.1\left(1-\frac{x}{500}\right) - 5y\right)$$

$$\frac{dy}{dt} = -.2y + .0005xy = y(-.2 + .0005x)$$

1. What happens to the prey population if there are no predators?

If $y(t)=0$, then the first equation becomes $\frac{dx}{dt} = .1x\left(1-\frac{x}{500}\right)$. This is logistic growth with carrying capacity $K = 500$

2. What happens to the predator population if there are no prey?

If $x(t)=0$ the second equation becomes $\frac{dy}{dt} = -.2y$. So the population dies off at an exponential rate.

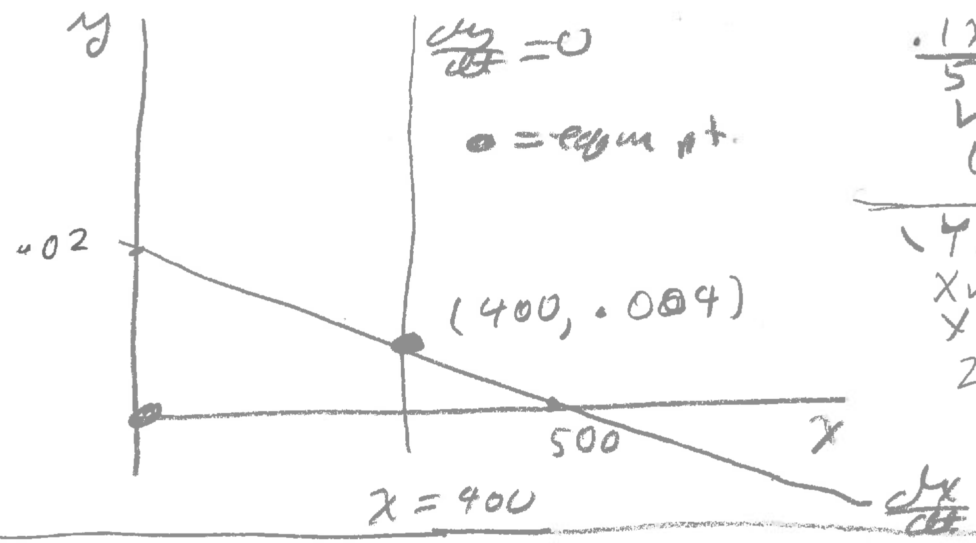
3. Draw the phase space and do an analysis of what happens when both predator and prey are present.

<u>eqm points</u> $\frac{dx}{dt}=0 \Rightarrow x=0$ or $.1\left(1-\frac{x}{500}\right) - 5y = 0$

so $x=0$ or $\frac{.1x}{500} + 5y = .1$

$\frac{dy}{dt}=0 \Rightarrow y=0$ or $-.2 + .0005x = 0.$

so $y=0$ or $x = \frac{.2}{.0005} = 400 = \hat{x}$



$\frac{.1x}{500} + 5y = .1$ has intercepts $(500, 0)$, $(0, .02)$

$Y1 = (.1 - .1X/500)/5$

Xmin = 0
Xmax = 500
2nd calc 1:value
X = 400
Y = .004

$\boxed{\text{So in the long run the populations settle down to } \hat{x} = 400,\ \hat{y} = .004.}$