Mathematics 172

Quiz 32 Name: Key

You must show your work to get full credit.

1. In a large aquarium there is algae growing and also a population of snails which are eating the algae. Let $V(t)$ be the number or grams of algae in the aquarium on day t and $P(t)$ the number of grams of snails in the aquarium. Assume that these satisfy the predator-victim system of equations:

$$\frac{dV}{dt} = .5V - .2VP \quad = V(.5 - .2P)$$
$$\frac{dP}{dt} = -.8P + .02VP \quad = P(-.8 + .02V)$$

(a) If $V(0) = 38$ and $P(0) = 2$ find

$V'(0) =$ 3.8

$V'(0) = V(0)(.5 - .2P(0))$
$= 38(.5 - .2(2)) = 3.8$

$P'(0) =$ -.08

$P'(0) = P(0)(-.8 + .02V(0))$
$= 2(-.8 + .02(38)) = -.08$

(b) Estimate: $V(0.3) \approx$ 39.14

$V(0.3) \approx V(0) + V'(0)(.3 - 0)$
$= 38 + 3.8(.3)$
$= 39.14$

$P(0.3) \approx$ 1.976

$P(0.3) \approx P(0) + P'(0)(.3 - 0)$
$= 2 + (-.08)(.3) = 1.976$

(c) If $V(4) = 41$ and $P(4) = 3$ estimate:

$V(4.2) \approx$ 40.18

$V'(4) = 41(.5 - .2(3)) = -4.1$
$V(4.2) \approx 41 - 4.1(4.2 - 4) = 40.18$

$P(4.2) \approx$ 3.012

$P'(4) = 3(-.8 + .02(41)) = .06$
$P(4.2) \approx 3 + .06(4.2 - 4) = 3.012$

(d) What are the average sizes of the algae and snail populations?

$\widehat{V} =$

$-.8 + .02V = 0$
$\widehat{V} = \frac{.8}{.02} = 40$

$\widehat{P} =$ 2.5

$.5 - .2P = 0$
$\widehat{P} = \frac{.5}{.2} = 2.5$

(e) Draw the phase space showing complete with some loops and arrows showing which way things are moving.