Mathematics 172

Quiz 33 Name: Key

***You must show your work to get full credit.***

**1.** A new building for student housing is invaded by 5 cockroaches. Assume that the with no constraints the roach population doubles every week. Then how long until there are a billion roaches?

Let $N(t)$ = number of roaches after $t$ weeks

Time to 1,000,000,000 roaches. 27.57 weeks

$N(t) = 5\lambda^t$

$N(1) = 5\lambda^1 = 2 \cdot 5$

so $\lambda = 2$. $N(t) = 5(2)^t$

solve $5 \cdot (2)^t = 10^9$

$2^t = 10^9/5$

$t \ln(2) = \ln(10^9/5)$

$t = \ln(10^9/5)/\ln(2)$

$= 27.57$

**2.** Assume that $P'(t) = .15P(t)$ and $P(0) = 42$.

(a) Give a formula for $P(t)$. $P(t) =$ $42e^{.15t}$

we know that solution to $p' = rP$ is $P(t) = P(0)e^{rt}$

(b) What is the doubling time of $P(t)$? Doubling time is 4.62

solve

$P(t) = 2P(0)$

$42e^{.15t} = 2 \cdot 42$

$e^{.15t} = 2$

$.15t = \ln(2)$

$t = \ln(2)/.15 = 4.62$

**3.** Assume that 15 rabbits are released on an island that has no rabbits. Assume that this population grows exponentially and that a survey 3 years later finds there are 50 rabbits.

(a) Give a formula for the number, $N(t)$, of rabbits after $t$ years. $N(t) =$ $15(1.494)^t$

$N(t) = N(0)\lambda^t$. we are given $N(0) = 15$. so $N(t) = 15\lambda^t$.

we are also given

$N(3) = 50$ so

$15\lambda^3 = 50$

$\lambda^3 = 50/15$

$\lambda = (50/15)^{\frac{1}{3}}$

$= 1.494$

(b) What is the per capita growth rate of the rabbits? $r =$ .494 (rabbits)/rabbit

$r = \lambda - 1 = .494$

(c) Can this exponential growth hold indefinitely? Why?

No, if it did the population would become infinite, and would run out of resources.

4. A population grows according to the discrete logistic equation

$$N_{t+1} = N_t + .3N_t\left(1 - \frac{N_t}{200}\right) \qquad \text{and} \qquad N_0 = 150.$$

(a) Find the following

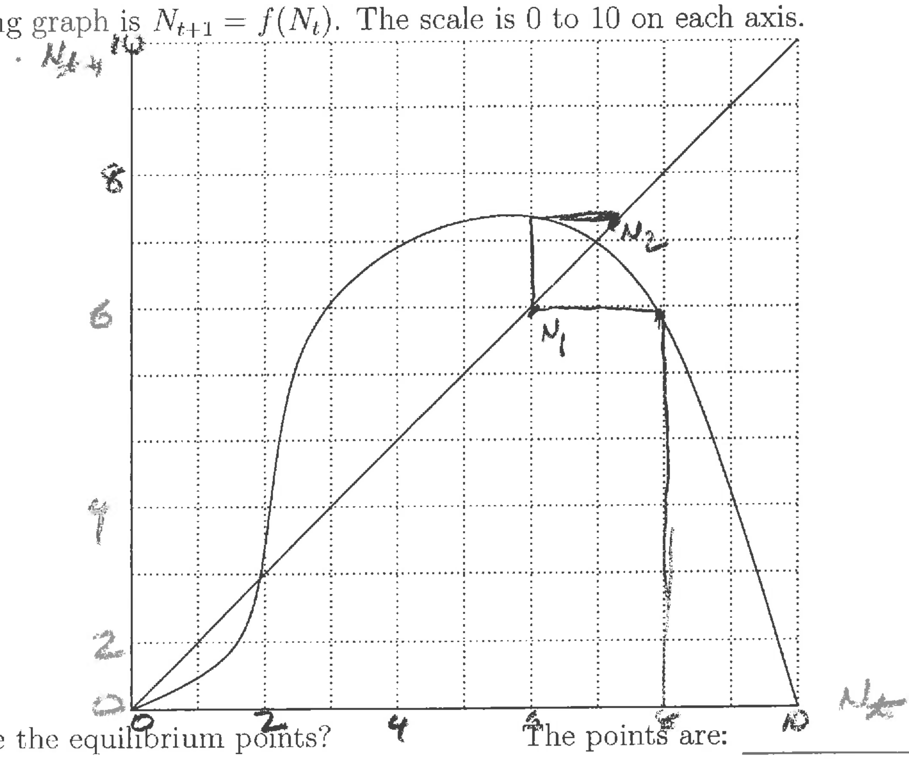
$N_1 =$ 161.25 $\qquad N_2 =$ 170.62 $\qquad N_3 =$ 178.14

(b) What is the carrying capacity? $\qquad K =$ 200

(c) Estimate the following

$N_{50} =$ 200 $\qquad N_{51} =$ 200 $\qquad N_{123} =$ 200

5. The following graph is $N_{t+1} = f(N_t)$. The scale is 0 to 10 on each axis.




(a) What are the equilibrium points? $\qquad$ The points are: 0, 2, 7

(b) Which of these points are stable? $\qquad$ Stable points are: 0, 7

(c) If $N_0 = 8$ estimate the following

$N_1 \approx$ 6 $\qquad N_2 \approx$ 7.3 $\qquad N_{100} \approx$ 7