Mathematics 172

Quiz 35 Name: Key

You must show your work to get full credit.

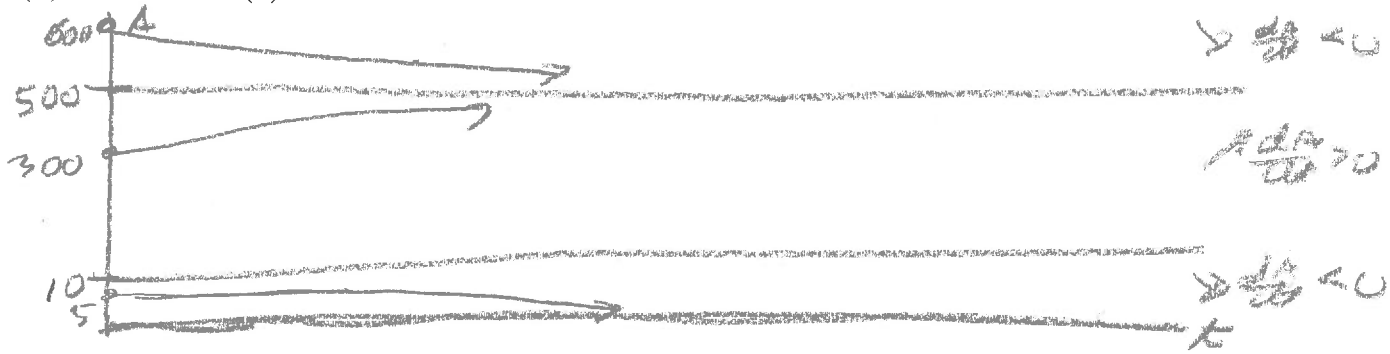
1. Water hyacinth is introduced into a pond. Let $N(t)$ be the number of pounds of it in the pond t weeks after in is introduced. Assume that A satisfies

$$\frac{dA}{dt} = .12A(A-10)(500-A)$$

(a) What are the equilibrium points of this rate equation?

Solve $\frac{dA}{dt}=0$ The equilibrium points are: 0, 10, 500

(b) Sketch a graph showing the equilibrium solutions and also the solutions with $A(0) = 5$, $A(0) = 300$ and $A(0) = 600$.



(c) Which of the equilibrium points are stable and which are unstable:

The stable points are: 0, 500

The unstable points are: 10

(d) For the solution with $A(0) = 5$ estimate $A(85)$. $A(85) \approx$ 0

(e) For the solution with $A(0) = 300$ estimate $A(85)$. $A(85) \approx$ 500

2. Water fleas are breading in a bucket. To start assume this population grows logistically with an intrinsic growth rate of $r = .3$ (fleas/week)/flea and a carrying capacity of $K = 200$.

(a) Let $N(t)$ be the number of water fleas in the bucket in week t. What is the rate equation satisfied by $N(t)$?

The equation is: $\frac{dN}{dt} = .3N\left(1-\frac{N}{200}\right)$

(b) Assume that after the population of water fleas has reached its carrying capacity that a single ~~mosquito~~ fish is added to the bucket and it eats the water fleas at the constant rate of 7 fleas/week. What is the new rate equation satisfied by $N(t)$?

The equation is: $\frac{dN}{dt} = .3N\left(1-\frac{N}{200}\right) - 7$

(c) What is the new stable population size of the water fleas?

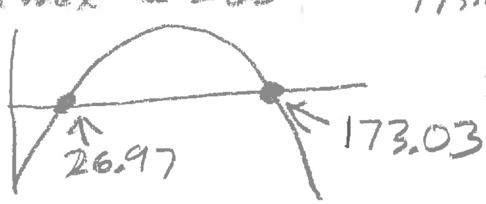
\Y1 = .3X(1 − X/200) − 7

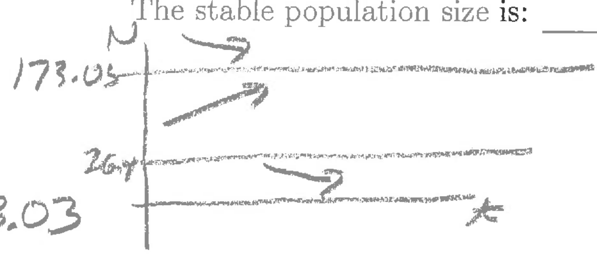
Xmin = 0

Xmax = 200

The stable population size is: 173.03

173.03 is stable





3. A population of fish is living in a polluted lake. Due to the pollution the intrinsic growth rate of the population is $r = -.08$ (fish/year)/fish. At what rate should the lake be stocked to have a stable population size of 5,000 fish?

Let $P(t)$ = size of population of fish in year t. Let

S = stocking rate in fish/year

Then

$$\frac{dP}{dt} = -.08P + S.$$

We want $P = 5000$ to be an equilibrium point

The stocking rate is: 400 fish/year

so use $\frac{dP}{dt} = 0$, $P = 5000$ in the equation to get

$0 = -.08(5000) + S$

so $S = (.08)(5000) = 400$

4. A cell has a volume of $V = 5.1 \times 10^{-6}\text{mm}^3$ and a surface area of $A = 7.4 \times 10^{-3}\text{mm}^2$. Assume that oxytgen, O_2, passes through the cell membrane at a rate of .4(mg/mm²)/hr.

(a) What is the total amount of O_2 coming into the cell per hour?

Amount per hours is: .00296 mg/hr

Total amount of O_2/hour

$= (\text{amount}/\text{Area}) \times \text{Area}$

$= (.4 \text{ mg}/\text{mm}^2) \times (7.4 \times 10^{-3} \text{ mm}^2)$

$= .00296$ mg/hr

(b) What is the amount of O_2 per volume coming into the cell per hour?

Amount of O_2 per volume per hour is; 580.34 (mg/mm³)/hr

$$\text{Amount}/\text{volume} = \frac{.00296 \text{ mg/hr}}{5.1 \times 10^{-6} \text{ mm}^3} = 580.39 \,(\text{mg}/\text{mm}^3)/\text{hr}$$

(c) If the cell needs 50(mg/mm²)/hr to live, then how much can can it be magnified before is dies from lack of oxygen?

Magnification factor is $\lambda =$ 11.6

If A_λ = magnified Area

V_λ = magnified volume

$A_\lambda = 7.4 \times 10^{-3} \lambda^2$

$V_\lambda = 5.1 \times 10^{-6} \lambda^3$

$$\text{Amount of } O_2/\text{volume} = \frac{(.4)(7.4 \times 10^{-3})\lambda^2}{5.1 \times 10^{-6}\lambda^3} = \frac{580.39}{\lambda} \left(\frac{\text{mg}}{\text{mm}^3}\right)/\text{hr}$$

So cut off is when $\frac{580.39}{\lambda} = 50$, $\lambda = \frac{580.39}{50} = 11.6$