Mathematics 172 Test 1 Nam

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 Points) A 10 inch carp weights .8 pounds. Use this information to estimate the weight of a 36 inch carp.

length of big carp = 36 in
length of small carp = 10 in
length of big carp = λ (length small carp)
36 in = λ (10 in)
$\lambda = \frac{36 \text{ in}}{10 \text{ in}}$
$\lambda = 3.6$

Estimated weight is 37.32 lbs

weight big carp = λ^3 (weight of small carp)
$W_{bc} = (3.6)^3 (.8 \text{ lbs})$
Weight of big carp = 37.32 lbs

2. (15 points) A population of squirrels grows exponentially starting with a population size of $P_0 = 15$ and with $P_1 = 17$.

(a) Give a formula for P_t. $P_t =$ $15(1.13)^t$

$P_t = P_0 (\lambda)^t$
$17 = 15(\lambda)^1$
$\lambda = \frac{17}{15}$ $\lambda = 1.13$

$P_t = 15(1.13)^t$

25
23
15
20
83

(b) What is the per capita growth rate? $r =$ .13

r − 1 = per capita growth rate
$1.13 - 1 = .13$

(c) How long until the population reaches 100 squirrels? $t =$ 15.52 years

$P_t = 15(1.13)^t$
$100 = 15(1.13)^t$
$\frac{100}{15} = (1.13)^t$
$\log(100/15)/\log(1.13) = t$
$t = 15.52$

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3. (15 points) A cell has volume $V = 5.1 \times 10^{-6}$ mm^3 and surface area 7.3×10^{-3}mm^2. Assume that oxygen, O_2, passes through the cell membrane at the rate of .38(mg/mm^2)/hr. If the cell needs 69(mg/mm^3)/hr to survive, then how much can it be magnified before it dies from lack of oxygen?

Magnification factor is $\lambda =$ 7.883

7.3×10^{-3} mm^2 $\cdot$ 0.38 (mg/mm^2)/hr = 0.002774 mg/hr

$V = 5.1 \times 10^{-6}$ mm^3 λ^3

$A = 7.3 \times 10^{-3}$ mm^2 λ^2

$O_2 = 0.002774$ mg/hr λ^2

$$\frac{0.002774 \text{ mg/hr } \lambda^2}{5.1 \times 10^{-6} \text{ mm}^3 \lambda^3} = \frac{543.922}{\lambda} \qquad \lambda = 7.883$$

$$\frac{543.922}{\lambda} = 69 \qquad \frac{543.922}{69} = \frac{69\lambda}{69}$$

4. (20 points) A national park that has not had wolves in it for 37 years has a population of ten wolves released. Assume that that population of wolves grows with a discrete logistic law with a per capita growth rate of $r = .25$ wolves/wolf and a carrying capacity of $K = 50$ wolves. Let P_t be the size of the wolf population in the park t years after they are released.

(a) Write down the discrete dynamical system satisfied by P_t.

$P_{t+1} = P_t + rP_t(1 - \frac{P_t}{K})$ $\quad P_{t+1} = P_t + (0.25)P_t(1 - \frac{P_t}{50})$ $\quad P_{t+1} =$ $P_t + 0.25 P_t (1 - \frac{P_t}{50})$

(b) What are P_1 and P_2?

$P_1 =$ 12

$P_2 =$ 14.28

$P_1 = 10 + 0.25(10)(1 - \frac{10}{50})$

$P_1 = 10 + 2.5(0.8)$

$P_1 = 12$

$P_2 = 12 + 0.25(12)(1 - \frac{12}{50})$

$P_2 = 12 + 3(0.76)$

$P_2 = 14.28$

(c) Estimate P_{30}.

$P_{30} \approx$ 50

Put in calculator

(or just use that in long run population will settle down to carrying capacity when $r < 2$ (so that K is stable)

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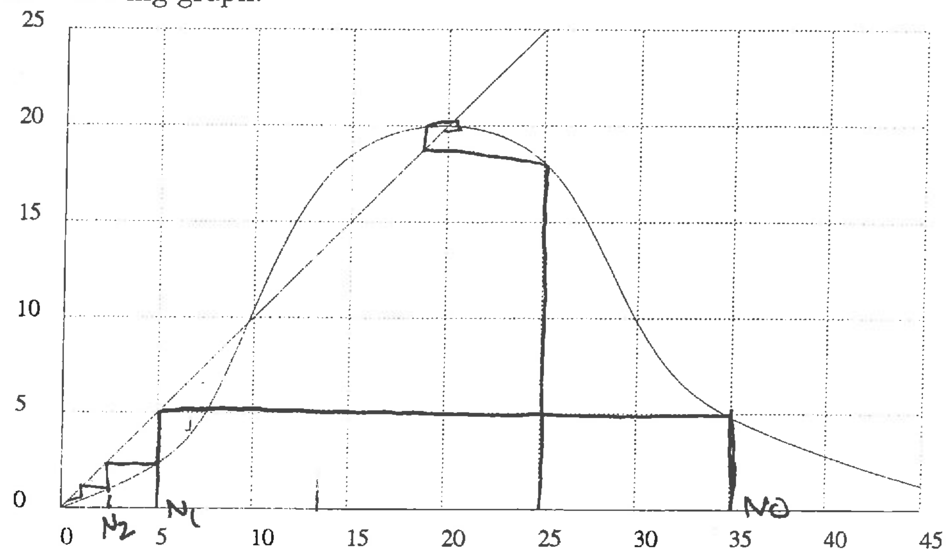
5. (20 points) My backyard has a pond that supports a population of frogs. Let N_t be the number of frogs t years after I first started counting them. Assume that

$$N_{t+1} = f(N_t)$$

where f has the following graph:



(a) What are the equilibrium points of this system?

Equilibrium points are 0, 10, 20

(b) Which of the equilibrium points are stable 0, 20

(c) Which of the equilibrium points are unstable 10

(d) If we start with $N_0 = 35$ frogs, then estimate N_1 and N_2.

$N_1 \approx$ 5 $N_2 \approx$ 2.5

(e) If we start with 5 frogs (that is $N_0 = 5$) estimate N_{50}. $N_{50} \approx$ 0

(f) If $N_0 = 25$ estimate N_{60}. $N_{60} \approx$ 20

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6. (20 points) Some squirrels are living on a small island. Let P_t be the size of the squirrel population in year t. Assume that if the population size in year t is P_t, the population size the next year is

$$P_{t+1} = P_t e^{.2(1-P_t/200)}$$

(a) If $P_0 = 180$ compute P_1 and P_2 accurate to 2 decimal places.

$P_1 = 180\, e^{0.2(1-180/200)} = 183.63624$

$P_2 = 183.63624\, e^{0.2(1-183.63624/200)} = 186.66594$

$P_1 =$ 183.64

$P_2 =$ 186.67

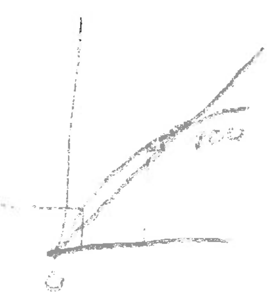
(b) Use your calculator to find the equilibrium points.

The equilibrium points are: 0, 200

The stable equilibrium points are: 200

The unstable equilibrium points are: 0

(c) Give a sentence or two explaining how you determined how the points were stable or unstable. (This explanation may involve computing some derivatives (i.e **dy/dx**) on the calculator.)



Two proofs:

First, the cobweb method shows that 200 is stable and 0 is not.

Second, $\frac{dy}{dx}$ for 200 is shown to be 0.8, while $\frac{dy}{dx}$ for 0 is 1.2214.

The rule for stability states that $\frac{dy}{dx}$ must be ≥ -1 or $\leq +1$; therefore 200 is stable but 0 is not.

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