Mathematics 172 Test 2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (5 points) What is the solution to the initial value problem

$$\frac{dA}{dt} = .2A \quad \text{and} \quad A(0) = 30.$$

Solution to $A' = rA$ is
$A(t) = A(0)e^{rt}$

$A(t) =$ $30e^{.2t}$

2. (10 points) Let $P(t)$ satisfy the logistic equation with inartistic growth rate $r = .05$ and $K = 100$.
(a) What is the rate equation for $P(t)$?

The rate equation is $\frac{dP}{dt} = .05P\left(1 - \frac{P}{100}\right)$

(b) If $P(4) = 90$ what is $P'(4)$? $P'(4) =$ $.45$

$P'(4) = \frac{dP}{dt} = .05P\left(1 - \frac{P}{100}\right)$
$= .05(90)\left(1 - \frac{90}{100}\right) = .45$

(c) If $P(4) = 90$ estimate $P(4.3)$. $P(4.3) \approx$ 90.135

$P(x) \approx P(a) + P'(a)(x-a)$
so
$P(4.3) \approx P(4) + P'(4)(4.3-4) = 90 + (.45)(.3) = 90.135$

(d) If $P(4) = 90$ estimate $P(100)$. $P(100) \approx$ 100

$t = 100$ is large so try then
the population should have reached the
carrying capacity. so $P(100) \approx K \approx 100$

3. (25 points) Let $P(t)$ satisfy the rate equation

$$\frac{dP}{dt} = .01P(P - 10)(20 - P)$$

(a) What are the equilibrium point(s)?

Solve $.01P(P-10)(20-P) = 0$ The equilibrium points are $0, 10, 20$

(b) Make a graph showing the equilibrium solutions and also the solutions with $P(0) = 5$, $P(0) = 15$ and $P(0) = 22$.

(c) Which of the equilibrium points are stable and which are unstable?

Stable points $0, 20$

Unstable points 10

(d) If $P(3) = 8$ estimate $P(3.5)$. $P(3.5) \approx$ 7.04

$P'(3) = .01(8)(8-10)(20-8)$
$= -1.92$

so
$P(3.5) \approx P(3) + P'(3)(3.5-3)$
$= 8 + (-1.92)(.5) = 7.04$

(e) If $P(3) = 8$, estimate $P(100)$. $P(100) \approx$ 0

starting at 8 we decrease down to 0. so for large values (say $t = 100$) $P(100) \approx 0$

(f) If $P(0) = 15$, estimate $P(100)$. $P(100) \approx$ 20

starting at 15 we increase up to 20. so $P(\text{large value}) \approx 20$. Thus $P(100) \approx 20$

4. (20 points) A population of mosquito fish lives in a small backyard pond. This population grows logistically with an intrinsic growth rate of $r = .5$ (fish/month)/fish and a carrying capacity of $K = 200$ fish. After the population of mosquito fish has reached its carrying capacity the owner adds a snakehead (a type of predatory fish) he has caught to the pond. The snakehead eats the mosquito fish at a rate of 15 fish/month. What is the new stable population size for the mosquito fish?

(a) Let $N(t)$ be the number of mosquito fish in the pond t months after the snakeheard is added. What is the rate equation satisfied by $N(t)$.

The rate equation is $\frac{dN}{dt} = .5N\left(1 - \frac{N}{200}\right) - 15$

(b) What is the new stable population of the mosquito fist?

To find equ. pts use calculator

Stable population size is 163.25 fish

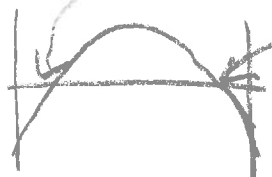
$Y1 = .5X(1 - X/200) - 15$

Xmin = 0

Xmax = 200

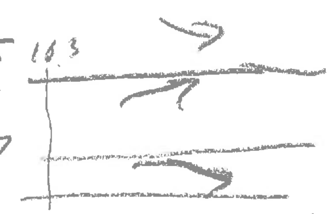
Zoom Fit

Use 2nd calc 2:zero



36.75

163.25



163

36.7

163.25 is stable

5. (15 points) An amusement park has as one of its attractions a wishing well. Guests are to throw pennies into the pond to get their wishes. The well a population of goldfish. Unfortunately copper is poisonous to fish so the population of declines exponentially with an intrinsic growth rate of $r = -.15$ (fish/week)/fish. Let $P(t)$ be the size of the goldfish population t weeks after the wishing well is opened.

(a) What is the rate equation satisfied by $P(t)$?

The rate equation is $\frac{dP}{dt} = -.15P$

(b) The management of the park wishes to have a stable population of 300 fish in the wishing well. At what rate should they stock it to have this happen?

Let S = stocking rate

The stocking rate is 45

Then we get a new rate equation

$$\frac{dP}{dt} = -.15P + S$$

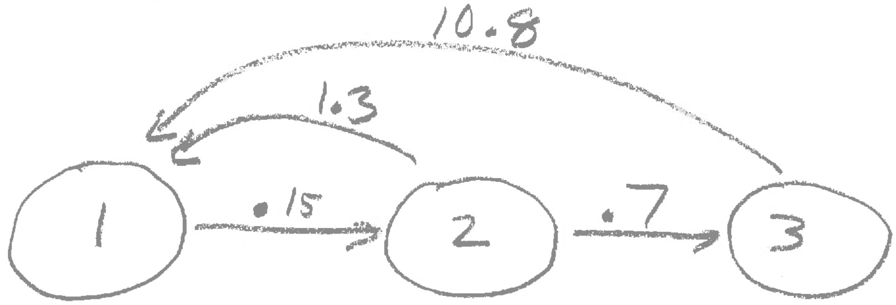
We want $P = 300$ to be an equ. pt. so $\frac{dP}{dt} = 0$

$$0 = -.15(300) + S \qquad S = .15(300) = 45$$

6. (25 points) A population of wild onions grows in a park. It has three stages: seedlings, juveniles, and adults. The Leslie matrix for this population is

$$L = \begin{bmatrix} 0 & 1.3 & 10.8 \\ .15 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$$

(a) Draw the loop diagram.



(b) What does the number 10.8 mean?

This is the averge number of offspring to do an adult that survive to be a seedling

(c) What does the number .7 mean? This is the proportion of juveniles that survive to be adults.

(d) If this year there are 200 plants in stage 1, 35 in stage 2, and 30 in stage 3 then find the following

Let $[A] = L = \begin{bmatrix} 0 & 1.3 & 10.8 \\ .15 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$

$[B] = \begin{bmatrix} 200 \\ 35 \\ 0 \end{bmatrix}$

Next year we have $[A][B] = \begin{bmatrix} 369.5 \\ 30 \\ 24.5 \end{bmatrix}$

The number in stage 1 next year 369.5

The number in stage 2 next year 30

The number in stage 3 next year 24.5

The number in stage 1 twenty years from now. 1852.7

The number in stage 2 twenty years from now. 277.4

The number in stage 2 twenty years from now. 168.1

20 years from now: $[A]^\wedge 20[B] = \begin{bmatrix} 1852.7 \\ 277.4 \\ 168.1 \end{bmatrix}$