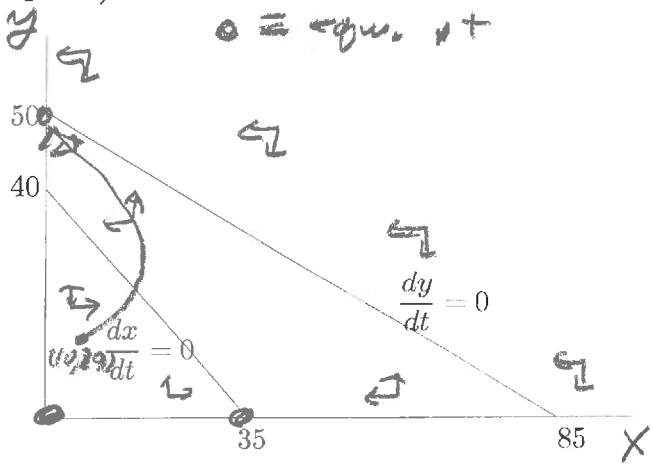
Mathematics 172 Test 3 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (30 points) This problem deals with a system of two competing species modeled by the equations

$$\frac{dx}{dt} = r_1 x\left(\frac{K_1 - x - \alpha y}{K_1}\right)$$
$$\frac{dy}{dt} = r_2 y\left(\frac{K_2 - \beta x - y}{K_2}\right)$$

(a)



If this is the phase space, find the following:

The x-species carrying capacity: 35

The y-species carrying capacity: 50

The equilibrium points: (0,0), (35,0), (0,50)

The stable points: (0,50)

If $x(0) = 10$ and $y(0) = 20$ estimate $x(100)$ and $y(100)$.

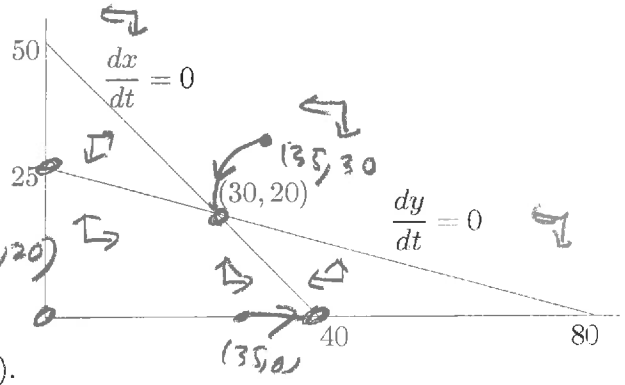
$x(100) \approx$ 0 $y(100) \approx$ 50

Circle one: x-species dominates,
y-species dominates,
competitive coexistence,
competitive exclusion.

0 = equ pt

(b)

If this is the phase space, find the following:



The x-species carrying capacity: 40

The y-species carrying capacity: 25

The equilibrium points: (0,0), (40,0), (0,25), (30,20)

The stable points: (30,20)

If $x(0) = 35$ and $y(0) = 30$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 30 $y(100) \approx$ 20

If $x(0) = 35$ and $y(0) = 0$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 40 $y(100) \approx$ 0

Circle one: x-species dominates,
y-species dominates,
competitive coexistence,
competitive exclusion.

2. (25 points) In a small pond a population of algae grows with a population of water fleas feeding on it. If x is the number of grams of algae and y is the number of grams of water fleas, assume they are related by the predator-prey system

$$\frac{dx}{dt} = .8x - .02xy = x(.8 - .02y)$$
$$\frac{dy}{dt} = -.6y + .12xy = y(-.6 + .12x)$$

where t is measured in weeks.

$x(0) = 4$

$y(0) = 42$

(a) If we start with 4 grams of algae and 42 grams of water fleas, compute $x'(0)$ and $y'(0)$.

$x'(0) =$ _____ $y'(0) =$ _____

$x'(0) = x(0)(.8 - .02x(0)) = 4(.8 - .02(42)) = -.16$

$y'(0) = y(0)(-.6 + .12y(0)) = 42(-.6 + .12(9)) = -5.04$

(b) Still assuming that we start with 4 grams of algae and 42 grams of water fleas, estimate $x(0.2)$ and $y(0.2)$.

$x(0.2) \approx$ 3.968 $y(0.2) \approx$ 40.992

$x(0.2) \approx x(0) + x'(0)(.2 - 0)$
$= 4 - (.16)(.2)$
$= 3.968$

$y(0.2) \approx y(0) + y'(0)(.2 - 0)$
$= 42 - 5.04(.4)$
$= 40.992$

(c) Find the average number of grams, $\widehat{x}$, and the average number of grams of water fleas, $\widehat{y}$.

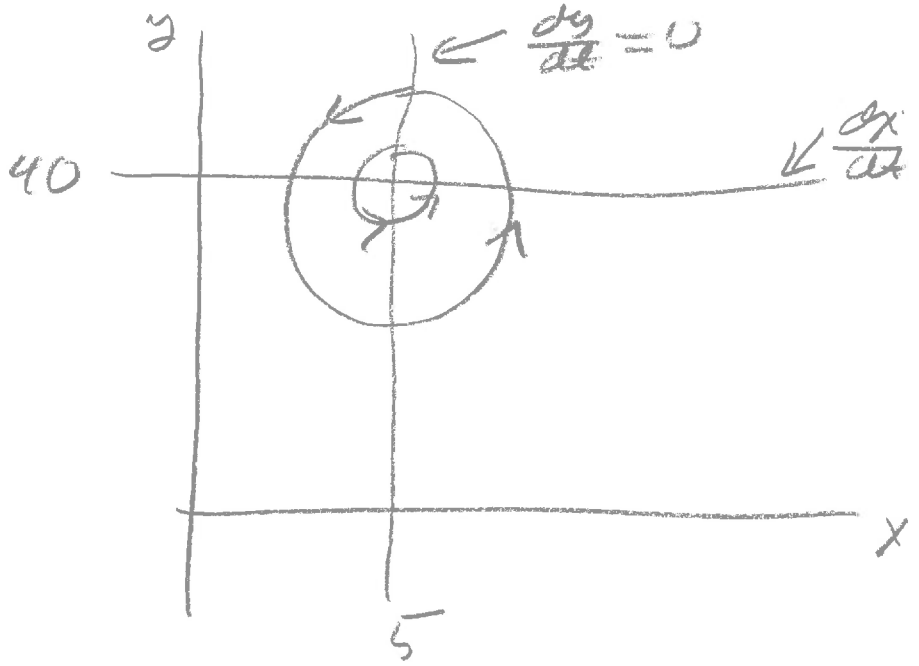
$\widehat{x} =$ 5 $\widehat{y} =$ 40

$(\widehat{x}, \widehat{y})$ is the nonzero equilibrium point.

From $\frac{dx}{dt} = 0$ we get $.8 - .02y = 0$ $y = \widehat{y} = \frac{.8}{.02} = 40$

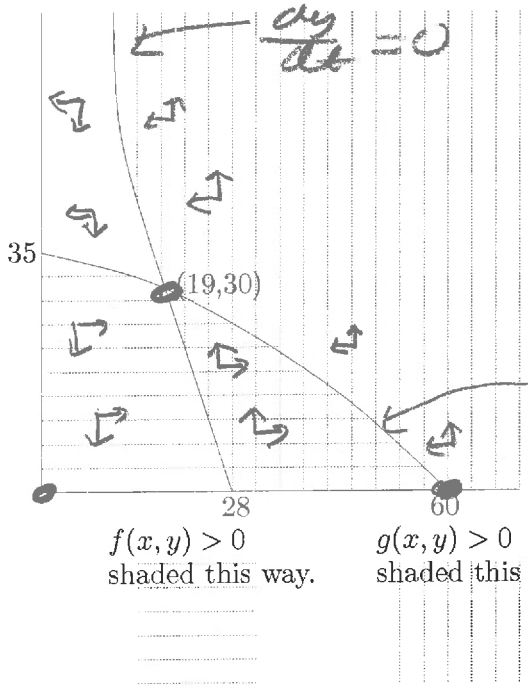
From $\frac{dy}{dt} = 0$ we get $-.6 + .12x = 0$ $x = \widehat{x} = \frac{.6}{.12} = 5$

(d) Draw the phase space complete with a couple of loops and arrows showing which way things are moving.



3. (15 points) Consider a system relating the sizes of two species, the x-species, and the y-species:

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y).$$



(a) Draw in the arrows showing the direction of motion.

(b) What are the equilibrium points.

Equilibrium points are: (0,0), (60,0), (19,30)

(c) If there is no y species present, then what is the carrying capacity for the x-species.

Carrying capacity is 60

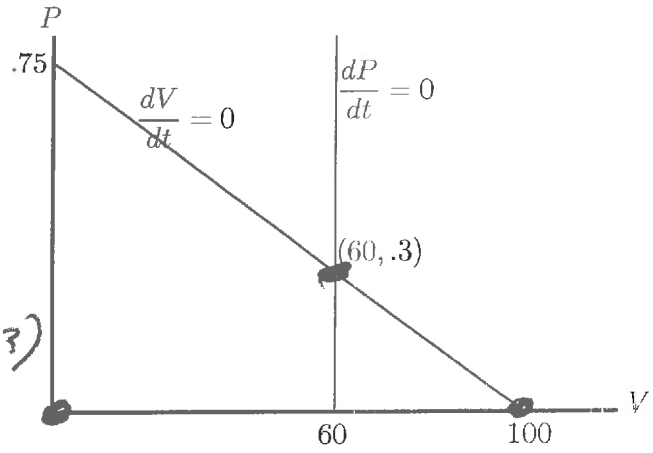
4. (30 points) This problem deals with a predator victim system modeled by the equations

$$\frac{dV}{dt} = rx\left(1 - \frac{V}{K}\right) - \alpha VP$$

$$\frac{dP}{dt} = -dP + \beta VP$$

• = equm pt.

(a)



If this is the phase space, find the following:

The vicitim carrying capacity: 100

The equilibrium points: (0,0), (100,0), (60,.3)

The stable points: (60,.3)

If $V(0) = 50$ and $P(0) = .4$ estimate $V(100)$ and $P(100)$.

$V(100) \approx$ 60 $P(100) \approx$ .3

If $V(0) = 50$ and $P(0) = 0$ estimate $V(100)$ and $P(100)$.

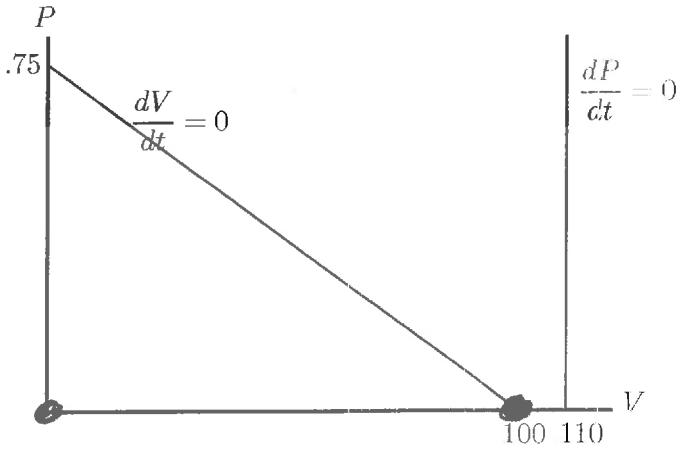
$V(100) \approx$ 100 $P(100) \approx$ 0

Write a couple of sentences explaining what happens in the long run (that is the two species coexist, or one the other or both die out.)

They coexists Due to the predator the carrying capacity of the victims is reduced to 60

• = equm. pt.

(b)



If this is the phase space, find the following:

The victim carrying capacity: 100

The equilibrium points: (0,0), (100,0)

The stable points: (100,0)

If $V(0) = 70$ and $P(0) = 10$ estimate $V(100)$ and $P(100)$.

$V(100) \approx$ 100 $y(100) \approx$ 0

Write a couple of sentences explaining what happens in the long run (that is the two species coexist, or one the other or both die out.)

The predator dies out as the prey population is not large enough to feed it.