Mathematics 172 Homework, February 27, 2019.

We saw in class today that for the Leslie matrix

$$L = \begin{bmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{bmatrix}$$

then the eigenvalue (which for us is the ***growth ratio*** of the stable population distribution) is the positive solution of the ***Euler-Lotka equation***

$$\frac{f_1}{\lambda} + \frac{p_1 f_2}{\lambda^2} + \frac{p_1 p_2 f_3}{\lambda^3} = 1.$$

Then the ***per capita growth rate*** of the stable population distribution is given by

$$r = \lambda - 1.$$

The stable age distribution is given by

$$\begin{bmatrix} \frac{1}{n} \\ \frac{p_1}{n\lambda} \\ \frac{p_1 p_2}{n\lambda^2} \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 \\ n_2 \\ n_3 \end{bmatrix}$$

where

$$n_2 = \frac{p_1}{\lambda}$$
$$n_3 = \frac{p_1 p_2}{\lambda^2}$$
$$n = 1 + n_2 + n_3$$

1. For the Leslie matrix

$$L = \begin{bmatrix} 0 & 3.1 & 23 \\ .05 & 0 & 0 \\ 0 & .9 & 0 \end{bmatrix}$$

(a) Find the growth ratio λ. *Solution:* By solving the Euler-Lotka equation on your calculator find that $\lambda = 1.06257$.

(b) What is the per capita growth rate of the stable distribution? *Solution:* It is $r = \lambda - 1 = .06257$.

(c) What is the stable age distribution? *Solution:* By plugging into the forumlas above we find:

Proportion in stage 1 = .920038

Proportion in stage 2 = .043293

Proportion in stage 3 = .036669

2. Compute the same things for the Leslie matrix

$$L = \begin{bmatrix} .08 & 5.6 & 49 \\ .02 & 0 & 0 \\ 0 & .85 & 0 \end{bmatrix}$$

Solution: Plug and chug gives

$$\lambda = 1.00908$$
$$r = .00908$$
$$\text{Proportion in stage } 1 = .96477$$
$$\text{Proportion in stage } 2 = .01912$$
$$\text{Proportion in stage } 3 = .01610$$