Mathematics 172 Homework, March 6, 2019.

Here are four example problems to look at.

1. For the system:

$$\frac{dx}{dt} = 0.10x\left(\frac{100 - x - 0.30y}{100}\right)$$
$$\frac{dy}{dt} = 0.20y\left(\frac{200 - 0.40x - y}{200}\right)$$

(a) Make a graph showing where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

Solution:

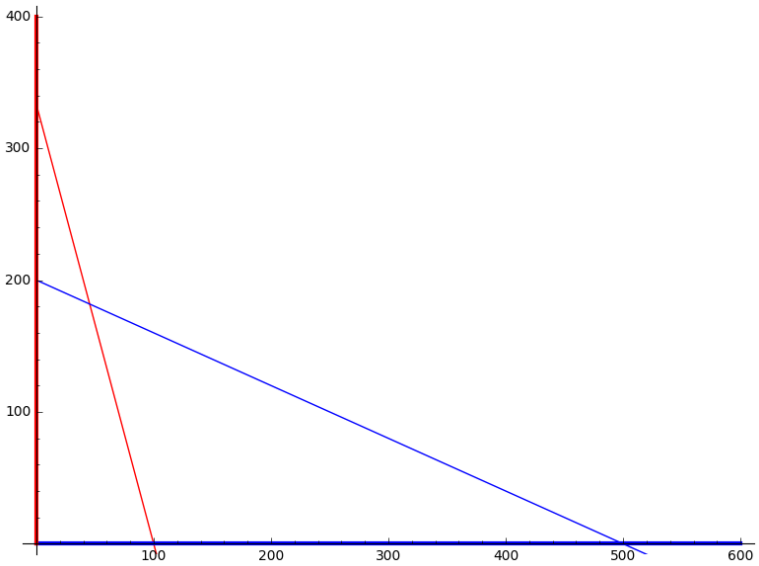

FIGURE 1. $\frac{dx}{dt} = 0$ is in red and $\frac{dy}{dt} = 0$ is in blue.

(b) What are the equilibrium points? *Solution:* The equilibrium points are (0,0), (100,0), (0,200), (45.455,181.818)

(c) What are the stable equilibrium points? *Solution:* The only stable point is $(45.455, 181.818)$.

2. For the system:

$$\frac{dx}{dt} = 0.10x\left(\frac{300 - x - 1.50y}{300}\right)$$
$$\frac{dy}{dt} = 0.20y\left(\frac{500 - 4.00x - y}{500}\right)$$

(a) Make a graph showing where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

Solution:

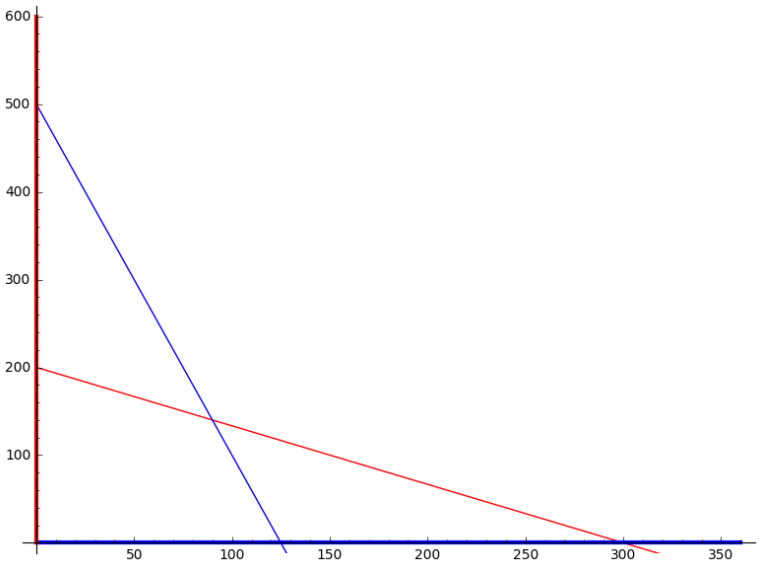

FIGURE 2. $\frac{dx}{dt} = 0$ is in red and $\frac{dy}{dt} = 0$ is in blue.

(b) What are the equilibrium points? *Solution:* The equilibrium points are (0,0), (300,0), (0,500), (90,140).

(c) Which of the equilibrium points are stable? *Solution:* The stable points are $(300, 0)$ and $(0, 500)$.

3. For the system:

$$\frac{dx}{dt} = 0.10x\left(\frac{300 - x - 1.50y}{300}\right)$$
$$\frac{dy}{dt} = 0.20y\left(\frac{500 - 1.20x - y}{500}\right)$$

(a) Make a graph showing where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

Solution:

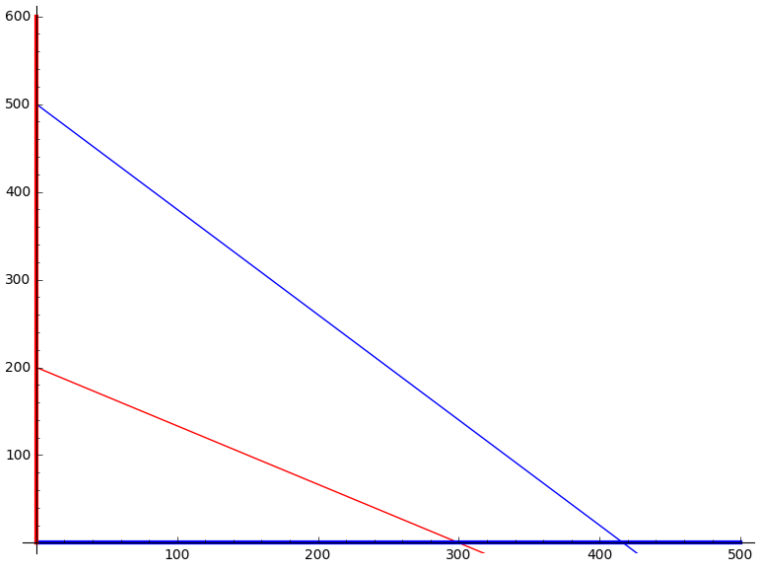

FIGURE 3. $\frac{dx}{dt} = 0$ is in red and $\frac{dy}{dt} = 0$ is in blue.

(b) What are the equilibrium points? *Solution:* The equilibrium points are (0,0), (300,0), (0,500).

(c) Which of the equilibrium points are stable? *Solution:* The only stable point is $(500, 0)$.

For the system:

$$\frac{dx}{dt} = 0.10x\left(\frac{500 - x - 1.20y}{500}\right)$$
$$\frac{dy}{dt} = 0.20y\left(\frac{300 - 1.60x - y}{300}\right)$$

(d) Make a graph showing where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

Solution:

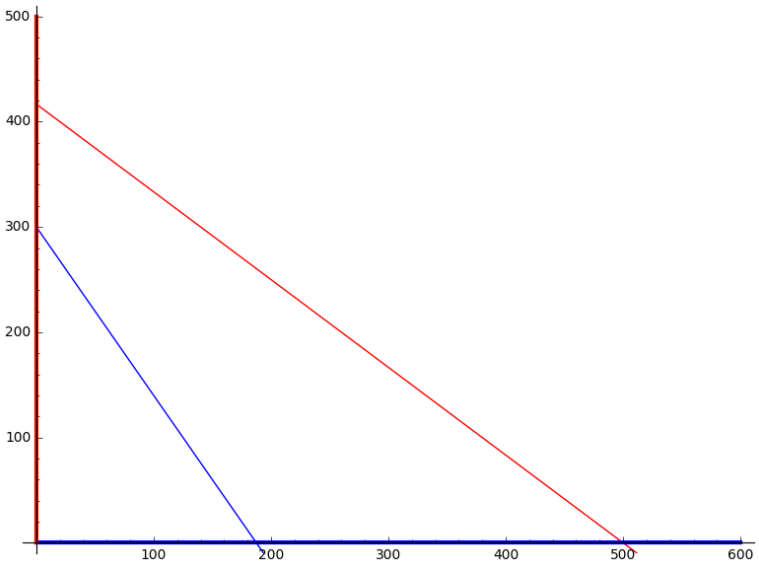

FIGURE 4. $\frac{dx}{dt} = 0$ is in red and $\frac{dy}{dt} = 0$ is in blue.

(e) What are the equilibrium points? *Solution:* The equilibrium points are (0,0), (500,0), (0,300).

(f) Which of the equilibrium points are stable? *Solution:* The only stable point is $(500, 0)$.