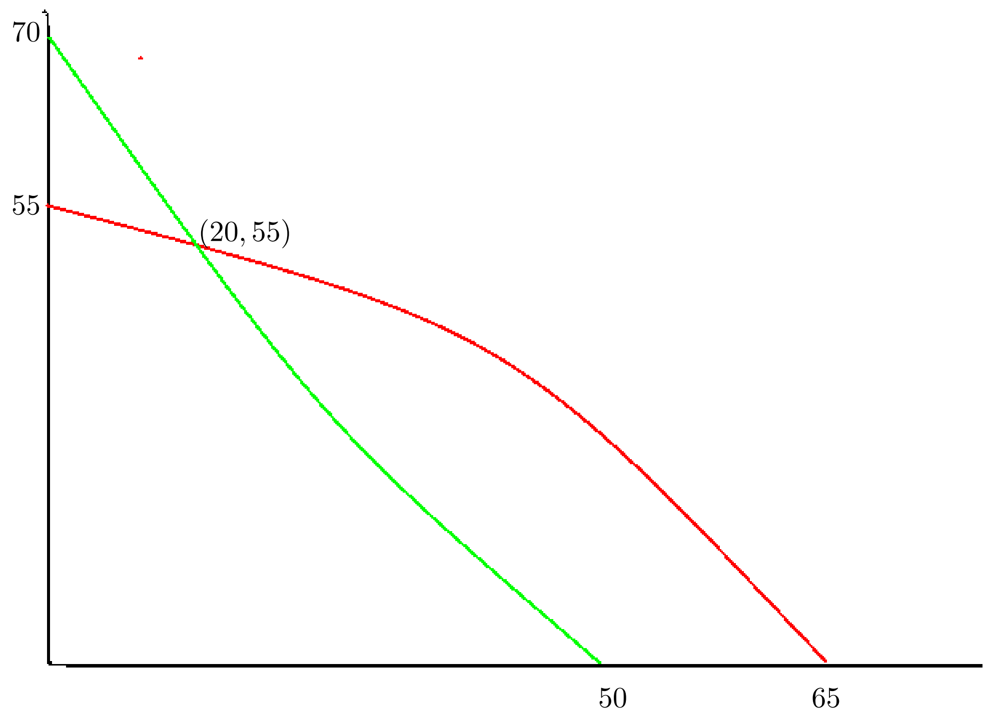
Mathematics 172 Homework, March 19, 2019.



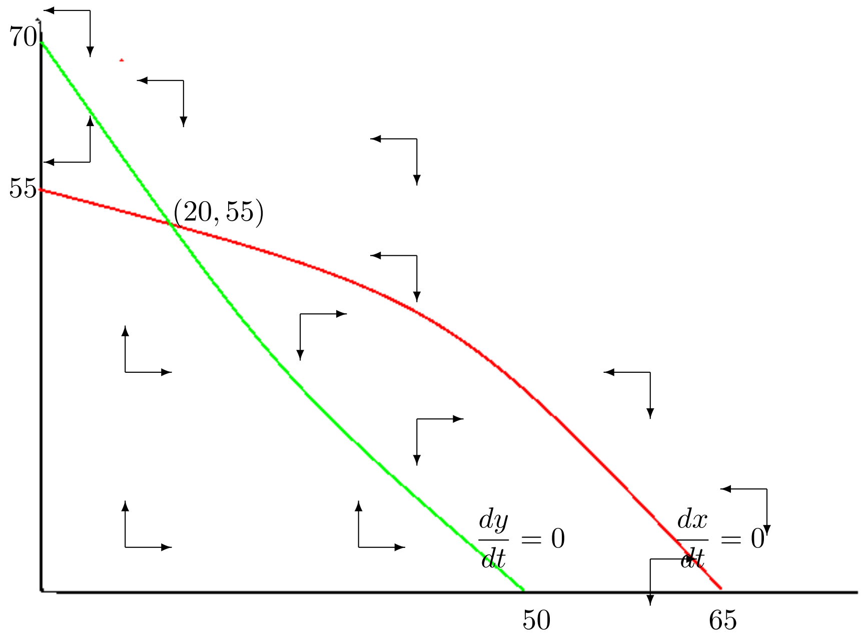
The figure above is the phase space for the rate equations

$$\frac{dx}{dt} = xf(x, y)$$
$$\frac{dy}{dt} = yg(x, y)$$

1. Assume that the red curve is where $f(x, y) = 0$ and the green curve is where $g(x, y) = 0$. Also assume $f(x, y) > 0$ for points below the red curve and $g(x, y) > 0$ for points below the green curve.

(a) Find all the rest points. *Solution:* The rest points are $(0, 0)$, $(65, 0)$, $(0, 70)$ and $(20, 55)$.

(b) Draw in the arrows in the different regions showing the direction that a point will move.



(c) Can you tell which of the rest points are stable or unstable? What about long term behavior? *Solution:* The rest points $(65, 0)$ and $(0, 70)$ are

stable. The other two are unstable. The long term behavior is ***competitive exclusion***.

(d) If there is a stable x population, is it possible for a small number of the y-species to invade the region? *Solution:* No the y-species can not

invade.

(e) If $x(0) = 5$ and $y(0) = 65$ estimate $x(100)$ and $y(100)$. *Solution:*

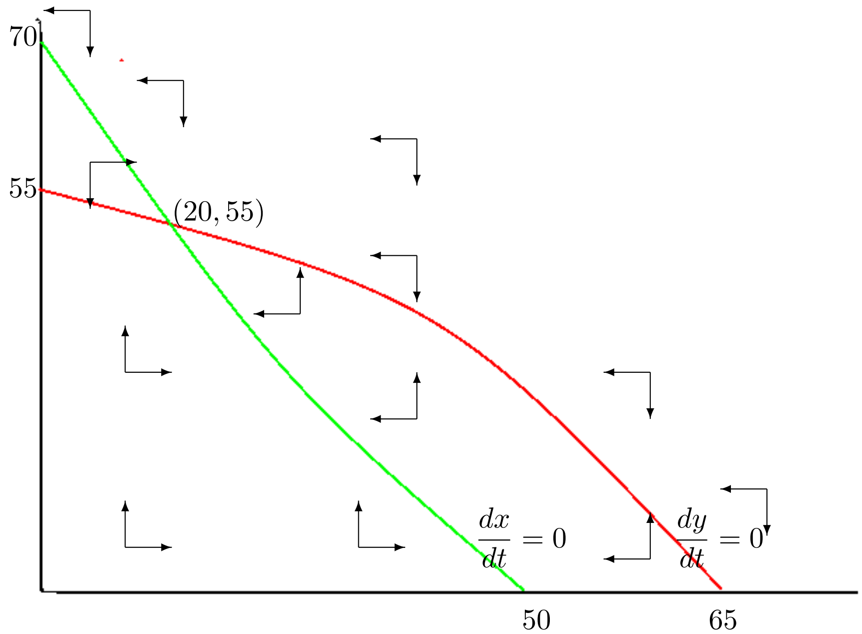
$x(100) \approx 0$ and $y(100) \approx 70$.

2. This time assume that assume that the red curve is where $g(x, y) = 0$ and the green curve is where $f(x, y) = 0$. Assume that $f(x, y) > 0$ for points below the green curve and $g(x, y) > 0$ for points below the red curve.

(a) Find all the rest points. *Solution:* The rest points are $(0, 0)$, $(50, 0)$,

$(0, 55)$, and $(20, 55)$.

(b) Draw in the arrows in the different regions showing the direction that a point will move.



(c) Can you tell which of the rest points are stable or unstable? What about long term behavior? *Solution:* The only stable rest point is $(20, 55)$. The others are all unstable. This is a case of ***competitive coexistence***.

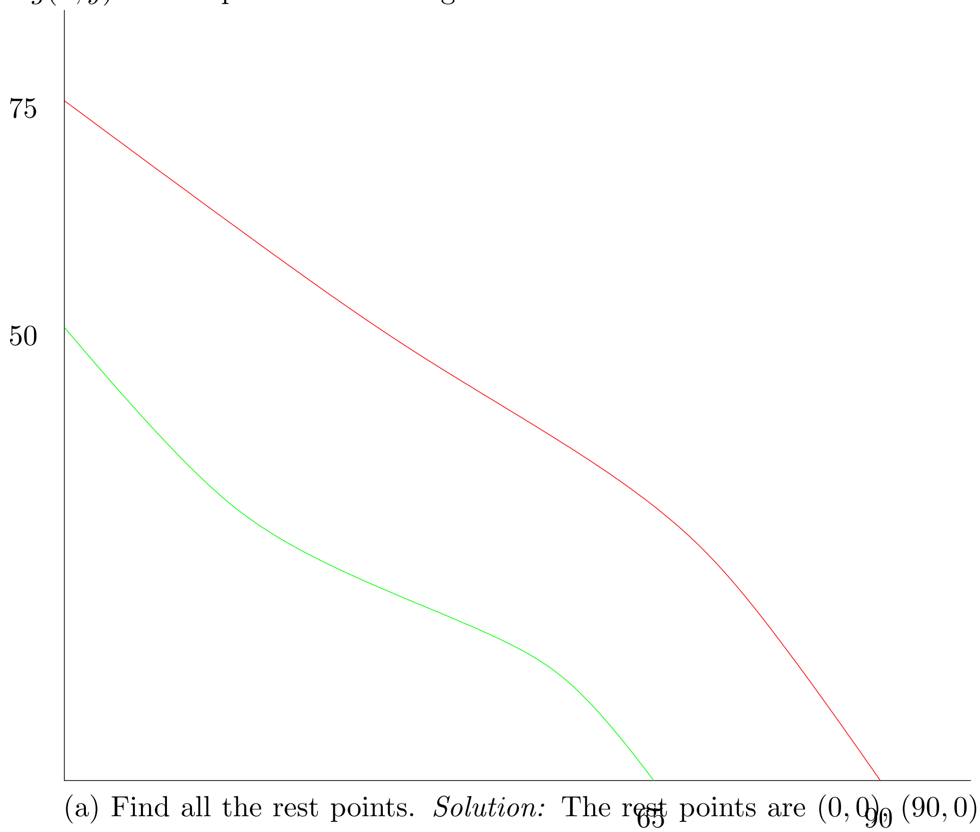
(d) If there is stable population of the y-species, then is a possible for a small number of the x-species to invade the region? *Solution:* Yes the x-species can invade.

(e) If $x(0) = 35$ and $y(0) = 45$, estimate $x(57)$ and $y(57)$. *Solution:* $x(57) \approx= 20$ and $y(57) \approx 55$.

3. We still consider a system:

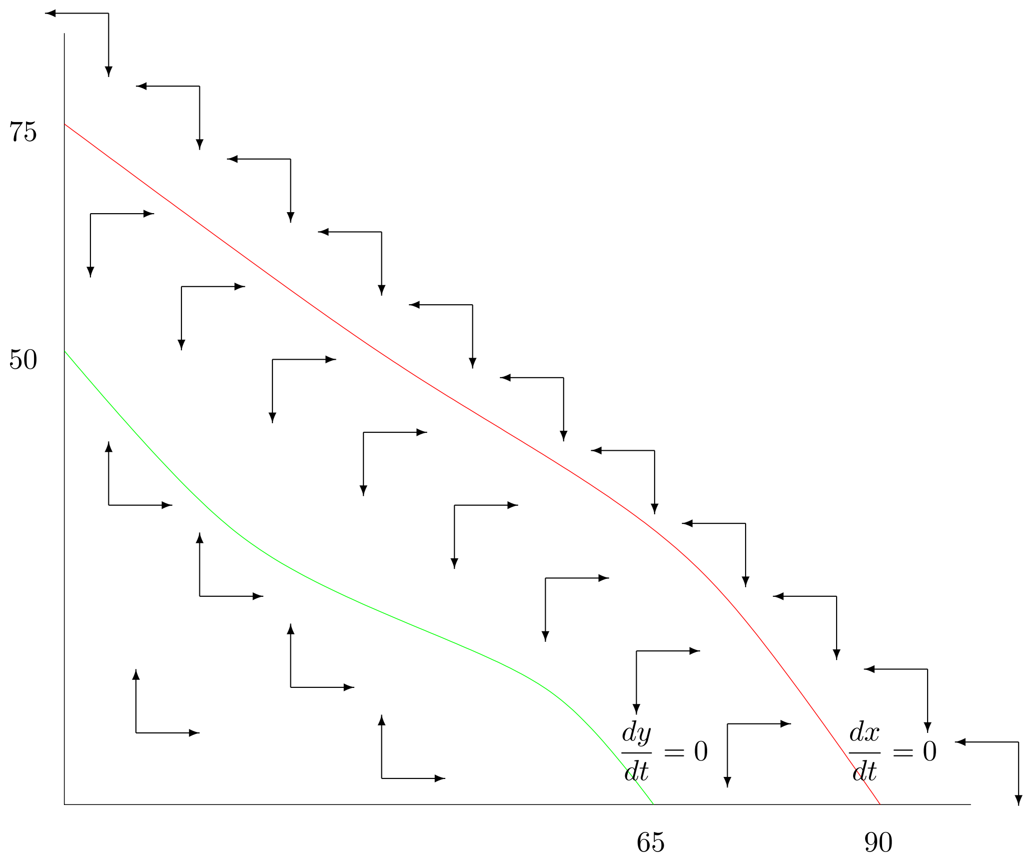
$$\frac{dx}{dt} = xf(x, y)$$
$$\frac{dy}{dt} = yg(x, y)$$

Assume that the red curve is where $f(x, y) = 0$ and the green curve is where $g(x, y) = 0$. Also assume $f(x, y) > 0$ for points below the red curve and $g(x, y) > 0$ for points below the green curve.



(a) Find all the rest points. *Solution:* The rest points are $(0, 0)$, $(90, 0)$, and $(0, 50)$.

(b) Draw in the arrows in the different regions showing the direction that a point will move.



(c) Which are the stable rest points? What is the long term behavior of the system? *Solution:* The only stable rest point is $(90, 0)$. The long term behavior is that the x***-species dominates***.

(d) If there is a stable population of the x-species, is it possible for the region to be invaded by a small number of the y-species? *Solution:* No the y-species can not invade.

(e) If there is a stable population of the y-species, is it possible for the region to be invaded by a small number of the x-species? *Solution:* Yes. In fact if there is a stable population of the y species and a small number of the x-species is introduced, then the x-species will dominate and drive the y-species to extinction.

(f) If $x(0) = 5$ and $y(0) = 85$, estimate $x(100)$ and $y(100)$. *Solution:* $x(100) \approx 90$ and $y(100) \approx 0$.