Mathematics 172 Homework, April 3, 2019.

A commercial hog farm has 1,000 pigs that are keep together. At some point a virus starts in the population of pigs. Assume that the following SIR model gives a description of the progress of the virus

$$\begin{aligned} S' &= -.001SI \\ I' &= .001SI - .2I \\ R' &= .2I \end{aligned}$$

1. For those of you how did not get full credit on the quiz on using Euler's method, do three Euler step of length 1 starting with the values $S(0) = 990$, $I(0) = 10$, and $R(0) = 0$. *Solution:* The result of doing this is

$S(1) \approx 980.10$	$I(1) \approx 17.90$	$R(1) \approx 2.00$
$S(2) \approx 962.56$	$I(2) \approx 31.86$	$R(2) \approx 5.58$
$S(3) \approx 931.89$	$I(3) \approx 56.16$	$R(3) \approx 11.95$

2. What is the ***contact number***, which we have also called ***basic reproduction number***? *Solution:* It is

$$c = \frac{.2}{.001} = 200$$

3. Use the equation

$$I' = .001SI - .2I = I(.001S - .2)$$

to show that the maximum of I occurs when $S = 200$. *Solution:* The formula for I' shows that $I' = 0$ when $S = .1/.001 = 200$. Thus the maximum occurs when $S = 200$.

4. Recall that we showed in class that S and I are related by

$$I = I_0 - c(S - S_0) + c\ln(S/S_0),$$

where in our case $c = 200$. Since the maximum occurs when $S = c$ use this to show that the maximum of I is

$$I_{\max} = I_0 + (S_0 - c) + c\ln(c/S_0)$$

which in our case is

$$I_{\max} = I_0 + (S_0 - 200) + 200\ln(200/S_0).$$

5. Use the formula to find $I_{\max}$ in the following cases

(a) $S_0 = 999$ and $I_0 = 1$. *Solution:* $I_{\max} = 478.31$.

(b) $S_0 = 990$, $I_0 = 10$. *Solution:* $I_{\max} = 480.12$.

(c) $S_0 = 700$, $I_0 = 300$. *Solution:* $I_{\max} = 549.45$.

(d) $S_0 = 100$, $I_0 = 900$. *Solution:* $I_{\max} = 900$, which in not what the formula for $I_{\max}$ above gives. (It gives $I_{\max} = 938, 63$.) Why does the formula not apply, or if it does apply how do we interpret it?