

## Mathematics 172 Homework, April 15, 2019.

Let  $f$  and  $g$  be two functions. Then  $f$  and  $g$  are **propositional** if there is a constant  $c > 0$  such that

$$g = cf.$$

That is two quantities are proportional if one is a constant multiple of the other. The constant  $c$  is called the **constant of proportionality**.

A fact that we will use is that if  $f$  is proportional to  $g$  and  $g$  is proportional to  $h$ , then  $f$  is proportional to  $h$ .

1. If  $A$  is proportional to the square of  $L$  and  $A = 30$  when  $L = 5$ , then
  - (a) Find a formula for  $A$  in terms of  $L$ .
  - (b) If  $L = 20$  what is  $A$

*Solution:* (a)  $A = 1.2L^2$ , (b)  $A = 480$ .

2. Assume that the cost of wire is proportional to the length of the wire and then 40 feet of wire costs \$4.50, then what is the cost to 220 feet of wire?

*Solution:* It costs \$24.75.

3. Show that if  $u$  is proportional to  $v$  and  $v$  is doubled, then  $u$  doubled. Show that if  $v$  is tripled, then  $u$  is tripled.

*Solution:* If  $u$  is proportional to  $v$  then  $u = cv$

$$u = cv$$

for some constant  $c$ . Let  $u_o$  and  $v_o$  be the original values of  $u$  and  $v$ . Let  $u_n$  and  $v_n$  be the new values, that is the values after  $v$  is doubled. Then, just by what it means to double something, we have

$$v_n = 2v_o.$$

Now using the equation relating  $u$  and  $v$  get

$$u_n = cv_n = c(2v_o) = 2(cv_o) = 2u_o$$

That is the new value of  $u$  is double the original value of  $u$ .

In the case that the original value of  $v$  is tripled, then the argument is just the same, just replace the 2's in the above with 3's.  $\square$

4. Assume that  $A$  is proportional to the square of  $r$ , that is for some constant  $c$  we have  $A = cr^2$ . Show that if  $r$  is doubled, then the value of  $A$  is multiplied by 4. Show that if  $r$  is tripled, then the value of  $A$  is multiplied by 9.

*Solution:* If we denote that  $A$  depends on  $r$  by  $A(r)$ , then we have

$$A(r) = cr^2.$$

Doubling  $r$  means replacing  $r$  by  $2r$ . The effect on  $A$  is

$$A(2r) = c(2r)^2 = c2^2r^2 = c4r^2 = 4cr^2 = 4A(r).$$

That is the value of  $A(r)$  is multiplied by 4.

Tripling  $r$  is replacing  $r$  by  $3r$ . In this case we have

$$A(3r) = c(3r)^2 = c3^2r^2 = c9r^2 = 9cr^2 = 9A(r)$$

which shows that the value of  $A$  is multiplied by 9.  $\square$

Here are some problems related to what we did in class today. Recall that if we scale by a factor of  $\lambda$ . (That is we magnify by a factor of  $\lambda$ ), and we are measuring in meters then

Things with units of m	scale with a factor of $\lambda$
Things with units of $m^2$	scale with a factor of $\lambda^2$
Things with units of $M^3$	scale with a factor of $\lambda^3$
Weight	scales with a factor of $\lambda^3$

**5.** A male domestic cat is 9.5 inches in height and weights 10 lbs. The height of a large male Siberian tiger is 43 inches. What would be the weight of a male domestic cat if it is scaled up to a height to 43 inches? *Solution:* The weight of the rescaled cat would be 927.331 lbs. For comparison the weight of a large Siberian tiger is 661 lbs.

**6.** The largest snake in the fossile record is th titanoboa which lived about 60 million years ago is what is not Columbia South America. The largest of these were about 42 feet long. Currently the largest boa in the world is the anaconda. The largest anaconda measured to date was 17.09 feet long and weighted 215 pounds. Assuming that the titanoboa is a scaled up version of an anaconda estimate the weight of a 42 foot titanoboa. *Solution:* The estimated weight would be 3191.245 lbs. (According to Wikipedia this estimate is too high, meaning that the titanoboa had a slimmer build than the anaconda.

**7.** A cell has a width of  $W = 1.1 \times 10^{-6}$  m, a surface area of  $A = 7.26 \times 10^{-12}$  m<sup>2</sup> and a volume of  $V = 1.26445 \times 10^{-18}$  m<sup>3</sup>. If similarly shaped cell has a width of  $1.7 \times 10^{-6}$  m estimate its surface area and volume. *Solution:* The scaled surface are and volume are  $A \approx 1.734 \times 10^{-11}$  m<sup>2</sup> and  $V \approx 4.667 \times 10^{-18}$  m<sup>3</sup>.