Mathematics 172 Homework, April 24, 2019.

Let f and g be two functions. Then f and g are ***propositional*** if there is a constant $c > 0$ such that

$$g = cf.$$

That is two quantities are proportional if one is a constant multiple of the other. The constant c is called the ***constant of proportionality***.

A fact that we will use is that if f is proportional to g and g is proportional to h, then f is proportional to h.

1. If A is proportional to the square of L and $A = 30$ when $L = 5$, then
(a) Find a formula for A in terms of L.
(b) If $L = 20$ what is A

Solution: (a) $A = 1.2L^2$, (b) $A = 480$.

2. Assume that the cost of wire is proportional to the length of the wire and then 40 feet of wire costs \$4.50, then what is the cost to 220 feet of wire?

Solution: It costs \$24.75.

3. Assume that the area of a pizza is proportional to its diameter and that the cost is proportional to the area. Also assume that an 10 inch pizza costs 8 dollars.
(a) What is the cost of an 18 inch pizza?
(b) What is the cost of a 24 inch pizza?
(c) If a pizza costs \$100, then what is its diameter?

Solution: (a) \$25.92, (b) \$44.00, (c) The diameter is 35.355 in.

4. Show that if u is proportional to v and v is doubled, then u doubled. Show that if v is tripled, then u is tripled.

Solution: If u is proportional to v then $u = cv$

$$u = cv$$

for some constant c. Let u_o and v_o be the original values of u and v. Let u_n and v_n be the new values, that is the values after v is doubled. Then, just by what it means to double something, we have

$$v_n = 2v_o.$$

Now using the equation relating u and v get

$$u_n = cv_n = c(2v_o) = 2(cv_o) = 2u_o$$

That is the new value of u is double the original value of u.

In the case that the original value of v is tripled, then the argument is just the same, just replace the 2's in the above with 3's. $\square$

5. Assume that A is proportional to the square of r, that is for some constant c we have $A = cr^2$. Show that if r is doubled, then the value of A is multiplied by 4. Show that if r is tripled, then the value of A is multiplied by 9.

Solution: If we denote that A depends on r by $A(r)$, then we have

$$A(r) = cr^2.$$

Doubling r means replacing r by $2r$. The effect on A is

$$A(2r) = c(2r)^2 = c2^2r^2 = c4r^2 = 4cr^2 = 4A(r).$$

That is the value of $A(r)$ is multiplied by 4.

Tripling r is replacing r by $3r$. In this case we have

$$A(3r) = c(3r)^2 = c3^2r^2 = c9r^2 = 9cr^2 = 9A(r)$$

which shows that the value of A is multiplied by 9. □

Here are some problems related to what we did in class recently. Recall that if we scale by a factor of λ. (That is we magnify by a factor of λ), and we are measuring in meters then

Things with units of m	scale with a factor of λ
Things with units of m^2	scale with a factor of λ^2
Things with units of M^3	scale with a factor of λ^3
Weight	scales with a factor of λ^3

6. A male domestic cat is 9.5 inches in height and weights 10 lbs. The height of a large male Siberian tiger is 43 inches. What would be the weight of a male domestic cat if it is scaled up to a height to 43 inches? *Solution:* The weight of the rescaled cat would be 927.331 lbs. For comparison the weight of a large Siberian tiger is 661 lbs.

7. The largest snake in the fossil record is th titanoboa which lived about 60 million years ago is what is not Columbia South America. The largest of these were about 42 feet long. Currently the largest boa in the world is the anaconda. The largest anaconda measured to date was 17.09 feet long and weighted 215 pounds. Assuming that the titanoboa is a scaled up version of an anaconda estimate the weight of a 42 foot titanoboa. *Solution:* The estimated weight would be 3191.245 lbs. (According to Wikipedia this estimate is too high, meaning that the titanoboa had a slimmer build than the anaconda.

8. A cell has a width of $W = 1.1 \times 10^{-6}$ m, a surface area of $A = 7.26 \times 10^{-12}$ m^2 and a volume of $V = 1.26445 \times 10^{18}$ m^3. If similarly shaped cell has a width of 1.7×10^{-6} m estimate its surface area and volume. *Solution:* The scaled surface are and volume are $A \approx 1.734 \times 10^{-11}$ m^2 and $V \approx 4.667 \times 10^{-18}$ m^3.

Example 1. A cell has volume $V = 8 \times 10^{-6}\text{mm}^3$ and surface area $A = 3.6 \times 10^{-3}\text{mm}^2$. Assume that oxygen, O_2, passes through the cell membrane at a rate of $.5(\text{mg}/\text{mm}^2)/\text{hr}$

(a) What is the total amount of O_2 that is comming into the cell per hour?

Solution:

Total O_2/hour $= (3.6 \times 10^{-3}\text{mm}^2) \times .5(\text{mg}/\text{mm}^2)/\text{hr} = .0018\text{mg}/\text{hr}$.

(b) What is the amount of O_2 per volume comming into the cell per hour?

Solution: Take the last answer and divide by the volume:

$$\text{Rate of } O_2 \text{ per volume} = \frac{.0018\text{mg}/\text{hr}}{8 \times 10^{-6}\text{mm}^3} = 225(\text{mg}/\text{mm}^2)/\text{hr}.$$

(c) If the cell needs $50(\text{mg}/\text{mm}^3)/\text{hr}$ of O_2 to survive, then how much can it be magnified before it dies from lack of oxygen?

Solution: Let λ be the factor by which it is magnified. Then by our rules for scaling we have we have

$$V_{mag} = 8 \times 10^{-6}\lambda^3\text{mm}^3, \qquad A_{mag} = 3.6 \times 10^{-3}\lambda^2\text{mm}^2$$

Thus

$$\text{Total } O_2 \text{ intake} = A_{mag} \times .5(\text{mg}/\text{mm}^2)/\text{hr} = .0018\lambda^2\text{mg}/\text{hr}$$

and

$$\text{Rate of } O_2 \text{ per volume} = \frac{.0018\lambda^2\text{mg}/\text{hr}}{8 \times 10^{-6}\lambda^3\text{mm}^3} = \frac{225(\text{mg}/\text{mm}^2)/\text{hr}}{\lambda}.$$

The threshold where oxygen starvation sets in is when

$$\text{Rate of } O_2 \text{ per volume} = 50(\text{mg}/\text{mm}^3)/\text{hr}.$$

That is

$$\frac{225(\text{mg}/\text{mm}^2)/\text{hr}}{\lambda} = 50(\text{mg}/\text{mm}^3)/\text{hr}.$$

Solving for λ gives

$$\lambda = \frac{225}{50} = 4.5$$

Therefore the cell can only grow to 4.5 times its length. □

9. A cell has volume $V = 4.6 \times 10^{-6}\text{mm}^3$ and surface area $A = 6.7 \times 10^{-3}\text{mm}^2$. Assume that oxygen, O_2, passes through the cell membrane at a rate of $.62(\text{mg}/\text{mm}^2)/\text{hr}$

(a) What is the total ammount of O_2 that is comming into the cell per hour? *Answer:* $4.154 \times 10^{-3}\text{mg}/\text{hr}$.

(b) What is the amount of O_2 per volume comming into the cell per hour? *Answer:* $903.04(\text{mg}/\text{mm}^2)/\text{hr}$.

(c) If the cell needs $377(\text{mg}/\text{mm}^3)/\text{hr}$ of O_2 to survive, then how much can it be magnified before it dies from lack of oxygen?

Answer: The magnification factor is $\lambda = 18.06$.