Mathematics 172 Homework, January 23, 2019.

In class today we started working with rate equations. For the time being these will be equations of the form

$$\frac{dP}{dt} = f(P)$$

where $f(P)$ is some function of P. Let us look at some special cases. To start consider:

$$\frac{dP}{dt} = .5P(10 - P)$$

This equation tells use how to compute the derivative $P'(t)$ if we know $P(t)$. For example if $P(3) = 7$, then

$$P'(3) = .5P(3)(10 - P(3)) = .5(7)(10 - 7) = 10.5$$

Likewise if $P(13) = 11$ we have

$$P'(13) = .5P(13)(10 - P(13)) = .5(13)(10 - 13) = -19.5$$

Here are a couple of problems to practice this.

1. Let $N(t)$ satisfy

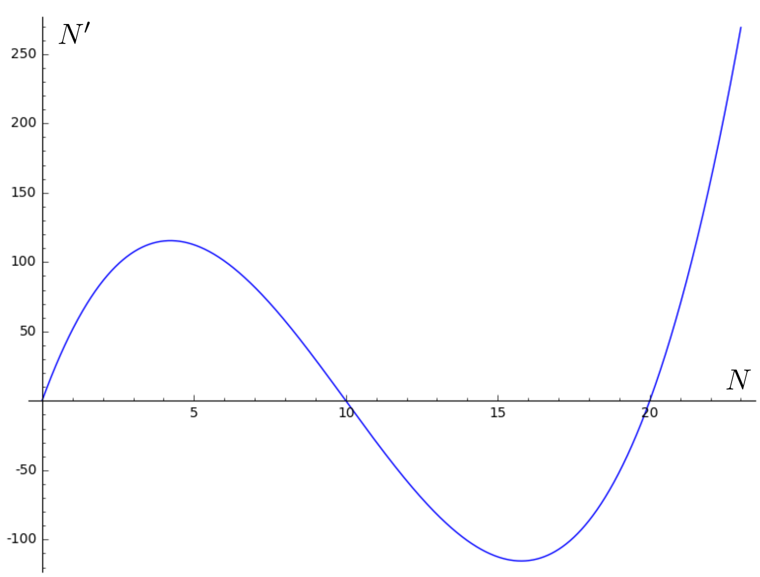
$$N'(t) = -.3N(N - 10)(N - 20) \tag{1}$$

(a) If $N(0) = 4$ what is $N'(0)$? *Solution:* $N'(0) = 115.2$.

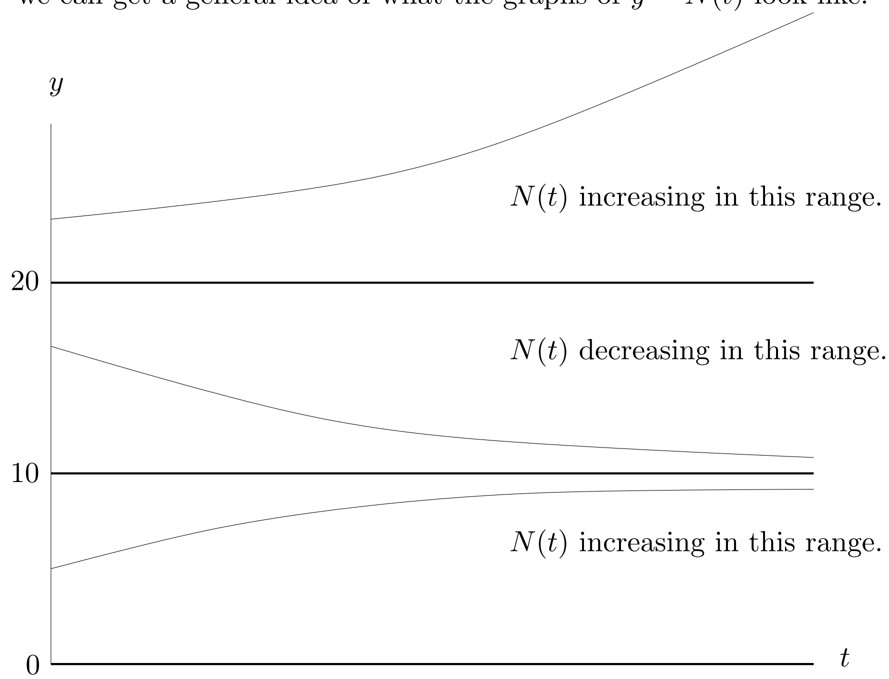
(b) If $N(5) = 21$ what is $N'(5)$? *Solution:* $N'(5) = 69.3$

(c) If $N(21.3) = 14.7$ what is $N'(21.3)$? *Solution:* $N'(21.3) = -109.8531$.

We remember from calculus that if the derivative of a function is positive, then the function is increasing and if the derivative is negative, then it is decreasing. Let us keep considering equation (1) for N'. Let us graph N' as a function of N. This will look like



What is important to us is that $N' > 0$ when $0 < N < 10$ and so N is increasing when N is in this interval. Likewise $N' < 0$ for $10 < N < 20$ and thus N is decreasing when $10 < N < 10$. Finally $N' > 0$ when $N > 20$ and so N is increasing on the interval $(0, \infty)$. This is enough information that we can get a general idea of what the graphs of $y = N(t)$ look like:



2. Let $N(t)$ be a solution to (1) with $N(0) = 5$. What is a reasonable estimate of $N(500)$? *Solution:* As $0 < N(0) < 10$ the solution with $N(0) = 5$

will be increasing. From the picture we see that $N(t)$ will stay below $N = 10$, but that it will keep getting closer to this value. Thus in the long run, for example when $t = 500$, the solution will have become very close to $N = 10$ and so $N(500) \approx 10$ is a good approximation.

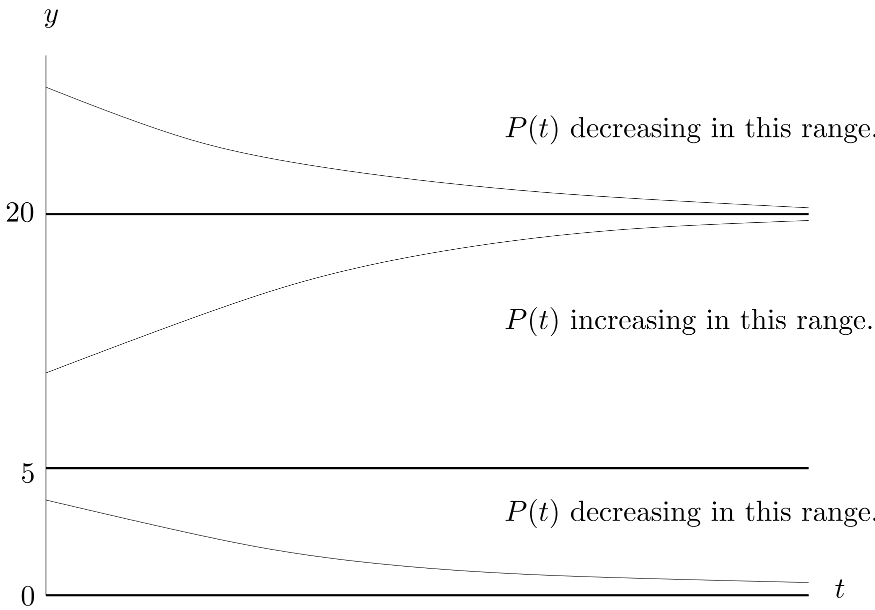
3. If $N(0) = 17$ estimate $N(432)$. *Solution:* The reasoning is just about

the same as in the previous problem. This time the function will be decreasing, but it will still be converging to $N = 10$ and so we still have the approximation $N(432) \approx 10$.

4. For the rate equation

$$\frac{dP}{dt} = -.3P(P-5)(P-15)$$

explain why the picture of solutions looks like:



and use this picture to explain to do the following

(a) If $P(0) = 4$ estimate $P(145)$.

(b) If $P(0) = 13$ estimate $P(215)$.

(c) If $P(0) = 24$ estimate $P(1{,}000)$.