Mathematics 172 Homework, January 25, 2019.

1. Assume that bass living a lake are overfished to that the intrinsic growth are of the population is $r = -.08$ (bass/bass)/year. Assume DNR (the Department of Natural Resources) stocks the lake at a constant rate of 5,000 bass/year.

(a) What is the rate equation satisfied by $P(t)$, the number of bass in the lake in year t? *Solution:*

$$\frac{dP}{dt} = -.08P + 5000.$$

(b) What is are the equilibrium solutions to this equation? *Solution:* Solve $-.08P + 5000 = 0$ to get that the only equilibrium solution is $P = 5000/0.08 = 62{,}500$ bass.

(c) Draw a picture of solutions that shows that 62,500 is a stable equilibrium solution.

2. Assume that hotel has a reflecting pond where it wishes to have a stable population of 500 goldfish. Due to the copper in the water from people using the pond as a wishing well, the intrinsic growth rate of the goldfish population is $r = -.05$ (fish/fish)/month. At what rate should the pond be stocked to keep the fish population at 500.

Solution: Let $N(t)$ be the number of goldfish in the pool after t months. Let S be the stocking rate for the pool (S is what we wish to solve for). The rate equation for the pollution size is then

$$\frac{dN}{dt} = -.05N + S.$$

We want $N = 500$ to be an equilibrium solutions. Putting $N = 500$ into the rate equation gives

$$0 = -.05(500) + S$$

Solving for S gives

$$S = .05(500) = 25 \text{ fish/month}$$

as the required stocking rate.