Mathematics 172

Quiz 15 **Name:** Key

You must show your work to get full credit.

This quiz is to show how populations converge to the stable age distribution. Let

$$L = \begin{bmatrix} 0 & 5.1 & 6.439 \\ 0.1 & 0 & 0 \\ 0 & .8 & 0 \end{bmatrix}$$

and

$$\vec{N} = \begin{bmatrix} 1.02 \\ 0.101 \\ 0.08 \end{bmatrix}$$

1. Compute $L\vec{N}$ $L\vec{N} =$ $\begin{bmatrix} 1.0302 \\ .1020 \\ .0808 \end{bmatrix}$

2. Compute $1.01\vec{N}$ $1.01\vec{N} =$ $\begin{bmatrix} 1.0302 \\ .1020 \\ .0808 \end{bmatrix}$

3. Write a sentence or two explaining why $\vec{N}$ is an eigenvector of L with eigenvalue $\lambda = 1.01$.

$L\vec{N} = \lambda\vec{N}$ to 4 dec. places and $L\vec{N} = \lambda\vec{N}$ is the definition of $\vec{N}$ being an eigenvector of L with eigenvalue λ

4. Use $\vec{N}$ to find the stable age distribution of L. Distribution is $\begin{bmatrix} .8493 \\ .0841 \\ .0666 \end{bmatrix}$

Now assume

$$\vec{N}(0) = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

5. Find $\vec{N}(5)$ and $\vec{N}(40)$ and their age distributions.

$\vec{N}(5) =$ $\begin{bmatrix} 5.2542 \\ .2601 \\ .4121 \end{bmatrix}$ Age distribution is $\begin{bmatrix} .8866 \\ .0439 \\ .0695 \end{bmatrix}$

$\vec{N}(30) =$ $\begin{bmatrix} 5.3862 \\ .5333 \\ .9224 \end{bmatrix}$ Age distribution is $\begin{bmatrix} .8493 \\ .0841 \\ .0666 \end{bmatrix}$