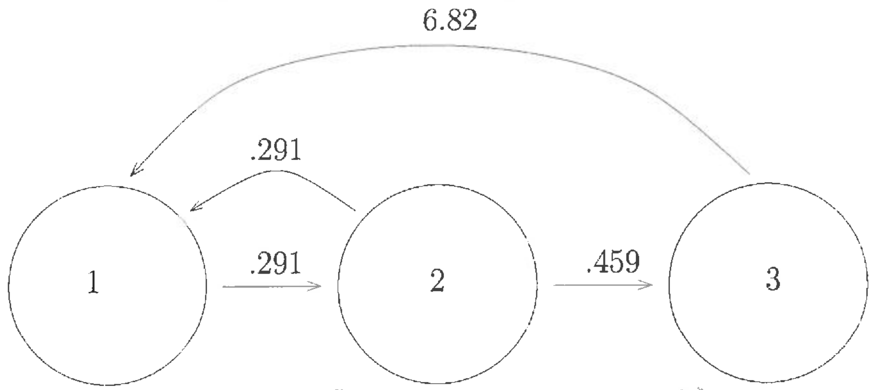
Mathematics 172

Quiz 23 Name: Key

You must show your work to get full credit.

1. A population of aquatic insects that has a two year life span (Stage 1 = larva, Stage 2 = juvenile, Stage 3 = adult) lives in a small pond. The life history is summarized by the loop diagram:



(a) What is the Leslie matrix? $L = \begin{bmatrix} 0 & .291 & 6.82 \\ .291 & 0 & 0 \\ 0 & .459 & 0 \end{bmatrix}$

This one

(b) What does the number 6.82 mean?

It is the average number of offspring produced by a stage 3 individual that live to stage 1

(c) What does the number .291 represent?

The proportion of stage one individuals that live to stage 2

(d) What proportion of larva survive to be adults? Proportion is .133569

$(.291)(.459) = .133569$

(e) If this year there are 75 larva, 22 juveniles, and 11 adults, then

Next year how many are in each stage?

Stage 1 81.422 Stage 2 21.825 Stage 3 10.098

After 5 years how many are in each stage?

Stage 1 74.89 Stage 2 23.44 Stage 3 10.05

After 20 what is the proportion of the population in each stage?

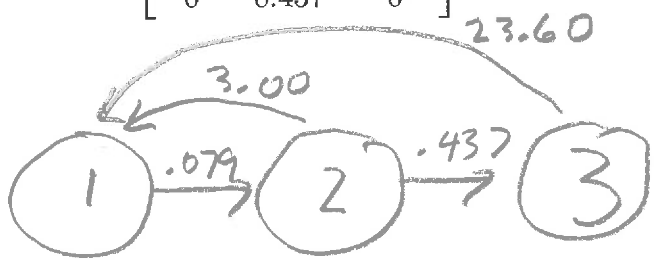
Stage 1 $\frac{73.65}{106.35} = .6925$ Stage 2 $\frac{22.67}{106.35} = .2132$ Stage 3 $\frac{10.03}{106.35} = .0943$

$\vec{N}_{20} = \begin{bmatrix} 73.65 \\ 22.67 \\ 10.03 \end{bmatrix}$ Total $= 73.65 + 22.67 + 10.03$
$= 106.35$

2. A population of the same type of insect in a different pond has Leslie matrix

$$\begin{bmatrix} 0.000 & 3.000 & 23.600 \\ 0.079 & 0 & 0 \\ 0 & 0.437 & 0 \end{bmatrix}$$

(a) Draw the loop diagram.



(b) You have access to a computer program that tells you that $\lambda = 1.02$ is an eigenvalue for this matrix and that the vector

$$\begin{bmatrix} 180 \\ 14 \\ 6 \end{bmatrix}$$

is an eigenvector. What are the following

The per capita growth rate. $r = \lambda - 1 = .02$

The stable age distribution is $\begin{bmatrix} .9 \\ .07 \\ .03 \end{bmatrix}$

An eigenvector is at the stable age distribution.

Total $= 180 + 14 + 6 = 200$ Stable distribution $= \begin{bmatrix} \frac{180}{200} \\ \frac{14}{200} \\ \frac{6}{200} \end{bmatrix} = \begin{bmatrix} .9 \\ .07 \\ .03 \end{bmatrix}$

3. For the initial problem

$$\frac{dy}{dt} = .1y\left(1 - \frac{y}{10}\right) \qquad y(0) = 7$$

(a) Use two steps of length .1 in Euler's method to estimate $y(.2)$.

$y(.2) \approx$ 7.0419

Step 1: $y'(0) = .1(7)(1 - \frac{7}{10}) = .21$

$y(.1) \approx y(0) + y'(0)(.1) = 7 + (.21)(.1) = 7.021$

Step 2: $y'(.1) \approx .1(7.021)(1 - \frac{7.021}{10}) = .2092$

$y(.2) \approx y(.1) + y'(.1)(.1) = 7.021 + (.2092)(.1) = 7.0419$

(b) Estimate $y(40)$. $y(40) \approx$ Carrying Capacity $= 10$

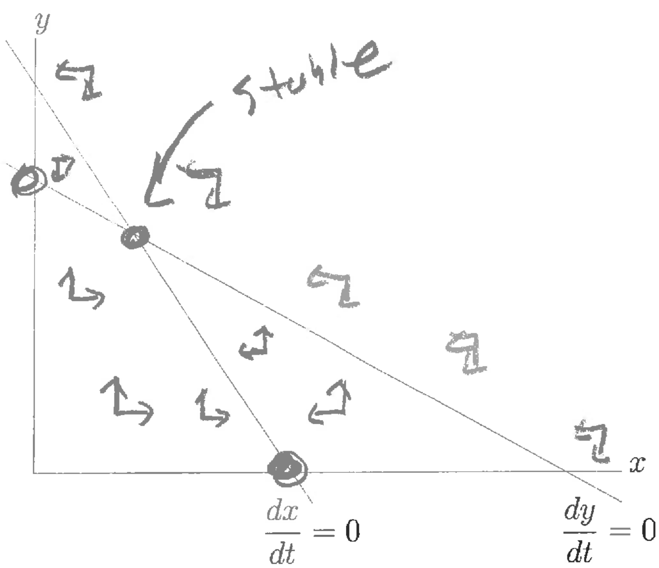
$y(40) \approx$ 10

4. The following are phase diagrams for the equations

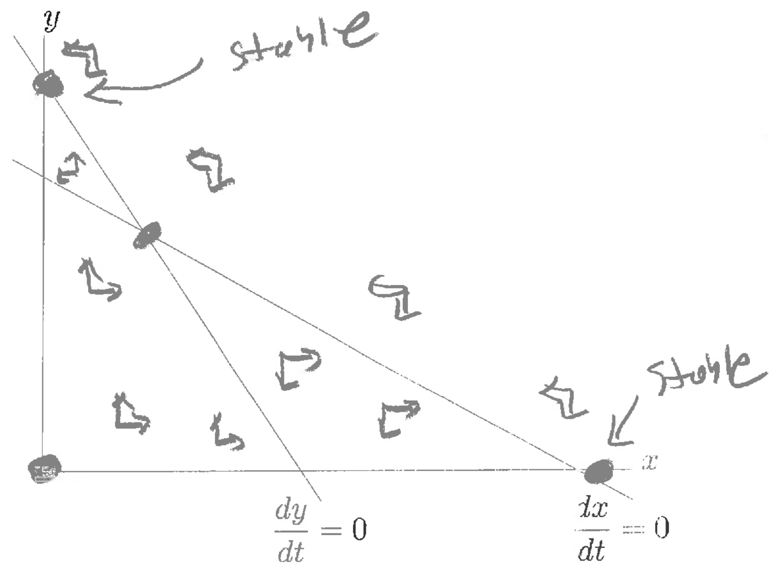
$$\frac{dx}{dt} = r_1 \left(\frac{K_1 - x - \alpha y}{K_1} \right)$$

$$\frac{dy}{dt} = r_2 \left(\frac{K_2 - \beta x - y}{K_2} \right)$$

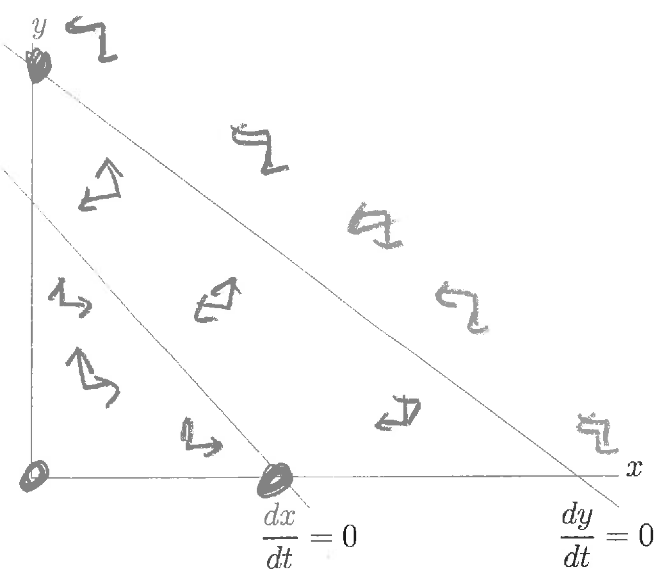
of competing species. In each of the figure label the rest points (or equilibrium points) with a large filled in circle ● and label which are stable. Also put in some arrows in each region showing which way the points (x, y) are moving. Also label as the long term behavior, this is if it is ***competitive coexistence*** or ***competitive exclusion***, ***x-species dominates***, or ***y-species dominates***.



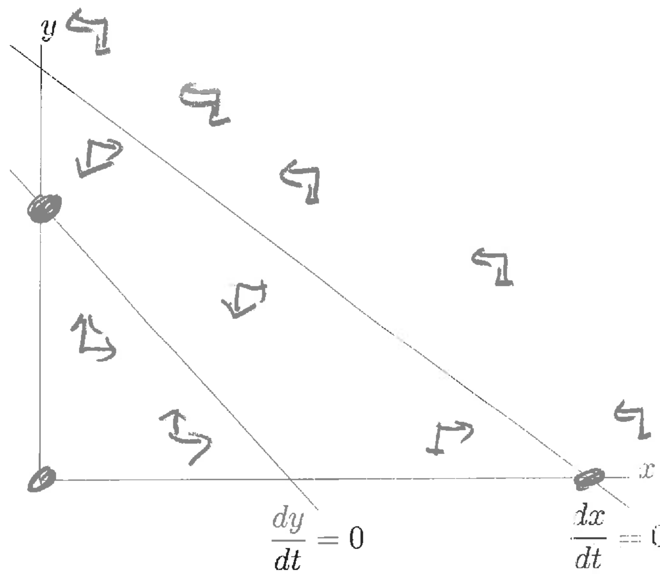
Long term behavior is: coexistence



Long term behavior is: exclusion



Long term behavior is: y-species dominates



Long term behavior is: x-species dominates

5. For the system of completing species

$$\frac{dx}{dt} = .1x\left(\frac{100 - x - 2y}{100}\right)$$

$$\frac{dy}{dt} = .15y\left(\frac{200 - 4x - y}{200}\right)$$

do this first

(a) Find the rest points. Rest points are $(0,0), (100,0), (0,200),$

$\frac{dx}{dt} = x\left(\frac{100-x-2y}{100}\right) = 0 \Rightarrow x = 0\ ,\ x+2y = 100$

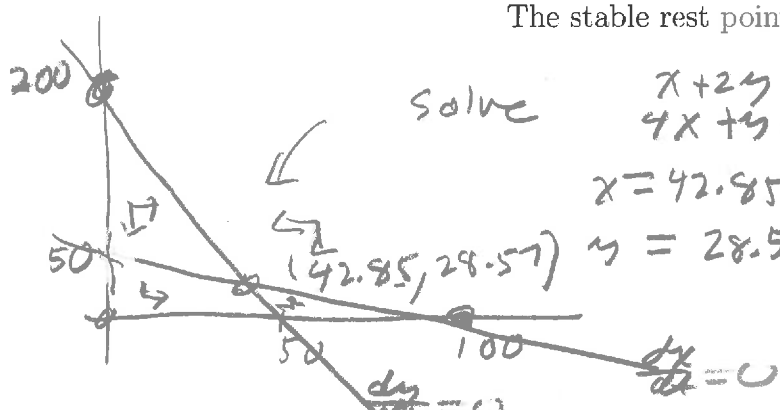
x-intercept $= (100,0)$
y-intercept $= (0,50)$

$\frac{dy}{dt} = y\left(\frac{200-4x-y}{200}\right) = 0 \Rightarrow y = 0 \quad 4x+y = 200$

x-intercept $= (50,0)$
y-intercept $= (0,200)$

(b) Graph the phase space and use it to classify which of the rest points are stable.

The stable rest points are: $(42.85, 28.57)$



Solve

$x + 2y = 100$
$4x + y = 200$

(used calculator
Y1 = (100 − X)/2
Y2 = 200 − 4X
2nd calc
5: intersect

$x = 42.85$
$y = 28.57$

(c) What is the long term behavior (circle one) ***Competitive constance***, ***Competitive exclusion***, ***x-species dominates***, or ***y-species dominates***.

(d) If there is a stable x population of 100, is it possible for a small number of the y-species to invade the region? Yes

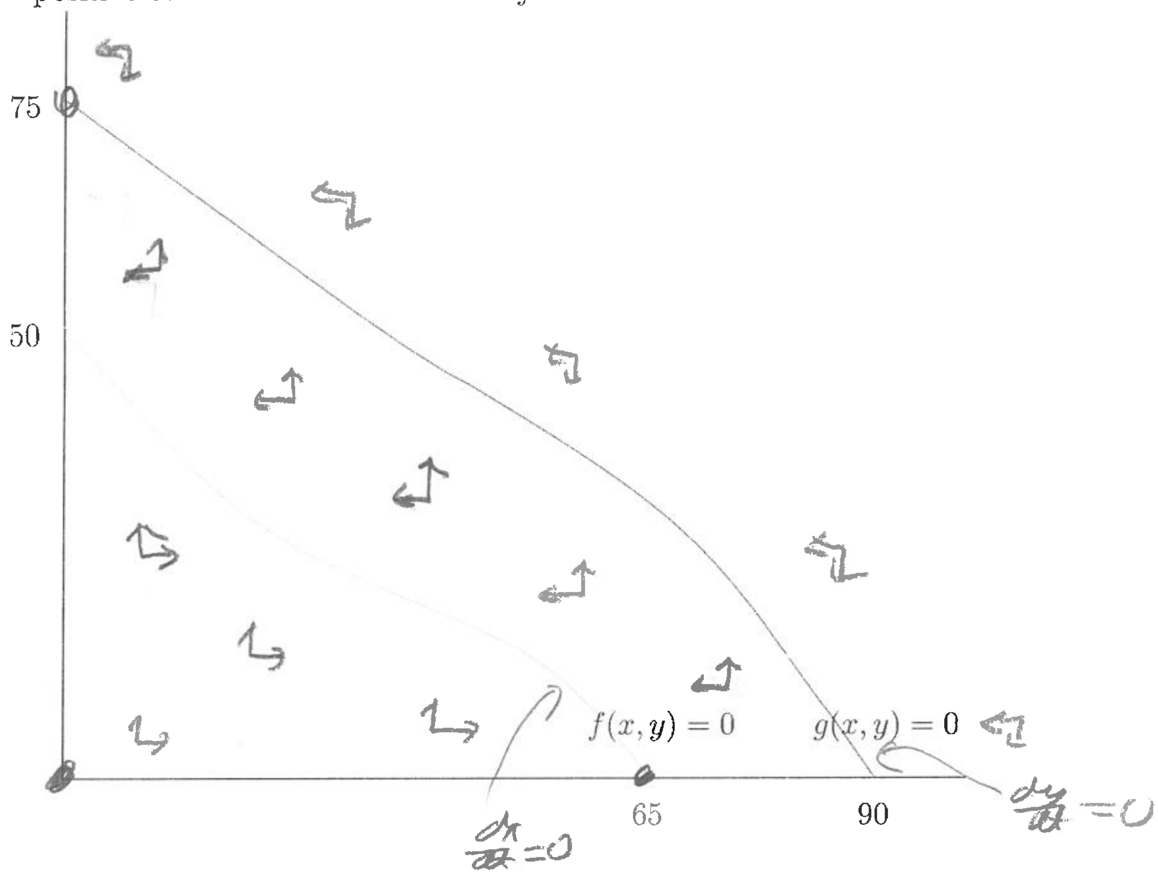
(e) If $x(0) = 5$ and $y(0) = 190$ estimate $x(95)$ and $y(95)$. It goes to the stable point

$x(95) \approx$ 42.85 $\qquad$ $y(95) \approx$ 28.57

6. For the system

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y)$$

assume that the curves $f(x,y) = 0$ and $g(x,y) = 0$ are as shown and that f both f and g are positive under the curves where they are zero:



(a) Find all the rest points. Rest points are: (0,0), (65,0), (0,75)

(b) Draw in the arrows in the different regions showing the direction that a point will move

(c) Which are the stable rest points? What is the long term behavior of the system?

The stable points are (0,75)

Long term behavior is y-species dominates

(d) If there is a stable population of the x-species, is it possible for the region to be invaded by a small number of the y-species?

(e) If there is a stable population of the y-species, is it possible for the region to be invaded by a small number of the x-species? NO.

(f) If $x(0) = 5$ and $y(0) = 85$, estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 0 $\qquad$ $y(100) \approx$ 75