Mathematics 172

Quiz 33

Name: Key

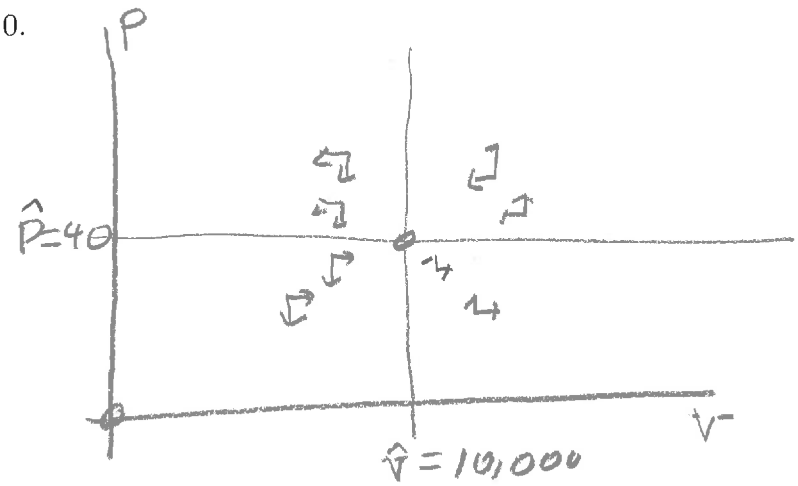
You must show your work to get full credit.

1. For the predator-victim system

$$\frac{dV}{dt} = .04V - .001VP = V(.04 - .001P) \qquad \hat{P} = \frac{.04}{.001} = 40$$

$$\frac{dP}{dt} = -3P + .0003VP = P(-3 + .0003V) \qquad \hat{V} = \frac{3}{.0003} = 10{,}000$$

(a) Draw the phase space space (V on the x-axis and P on the y-axis) and label the lines where $\frac{dV}{dt} = 0$ and $\frac{dP}{dt} = 0$.



(b) What are the are the average number of victims and predators?

$\widehat{V} =$ 10,000 $\widehat{P} =$ 40

(c) What happens to the average number of victims and predators if the death rate of the predator is halved?

New $\widehat{V} =$ 5,000 New $\widehat{P} =$ 40

This replaces 3 by 1.5 in the equation. Thus

$$\text{New } \hat{V} = \frac{1.5}{.0003} = 5{,}000$$

$$\text{New } \hat{P} = \text{old } \hat{P} = 40$$

2. For the predator-victim system

$$\frac{dV}{dt} = .2V\left(1 - \frac{V}{500}\right) - .03VP = V\left(.2\left(1-\frac{V}{500}\right) - .03P\right)$$

$$\frac{dP}{dt} = -4P + .004VP = P(-4 + .004V), \quad \hat{V} = \frac{4}{.004} = 1000$$

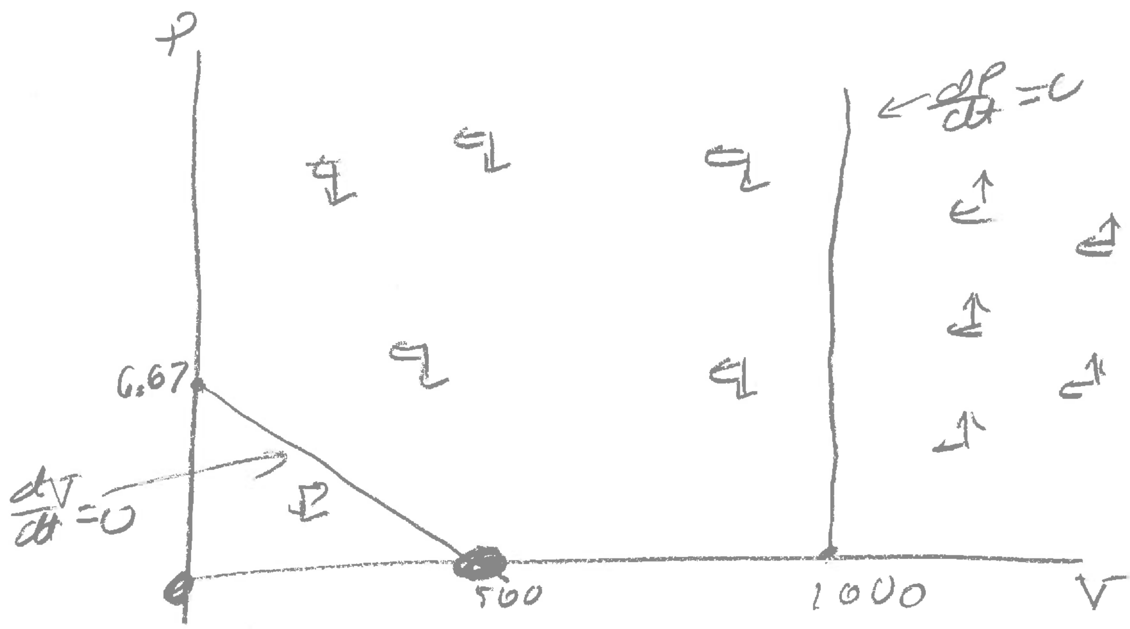
(a) What is the carrying capacity of the victim population if there are NO predators?

The first equation becomes

$\frac{dV}{dt} = .2V\left(1 - \frac{V}{500}\right)$ logistic with $K =$ 500

(b) Draw the phase plane of the system showing the rest points.

Rest points are (0, 0), (500, 0)



$\frac{dP}{dt} = 0$ on $P = 0$ and $V = 1000$

$\frac{dV}{dt} = 0$ on $V = 0$ and $.2\left(1 - \frac{V}{500}\right) - .03P = 0$

V intercept $V = 500$

P intercept $.2 - .03P = 0$

$P = \frac{.2}{.03} = 6.67$

3. For the SIR system

$$S' = -.0025SI$$
$$I' = .0025SI - .05I = I(.0025S - .05)$$
$$R' = .05I$$

(a) If $S(0) = 10$ does the infection start spreading (i.e is I increasing) or is it receding (i.e. is I decreasing)? Write a sentence or two explaining your answer.

$I' = I(.0025(10) - .05) = -.025I < 0$ so receding

(b) If $S(30) = 25$ is the infection spreading or receding? Write a sentence or explaining your answer.

$I' = I(.0025(25) - .05) = .0125I > 0$ so increasing

(c) What is the cut off size of S that determines if the infection is spreading or receding?

when $I' = I(.0025S - .05) = 0$ Cut off size is $S =$ 20

i.e. $S = \frac{.05}{.0025} = 20$

(d) What is the average duration of the infection?

This is $\frac{1}{.05} = 20$ The duration is 20 days

(e) If $S(5) = 90$, $I(5) = 10$ and $R(5) = 0$ do one step of length 1 in Euler's method to estimate $S(6)$.

$S'(5) = -.0025(90)(10) = 2.25$ $S(6) \approx$ 12.25

so

$S(6) \approx S(5+1) \approx S(5) + S'(5)(1)$

$= 10 + 2.25 = 12.25$

4. The longest snake in the fossil record is the *Titanoboa* which lived 60 million years ago in the South American country of Columbia, and was 42 feet long. We make the assumption that the *Titanoboa* was proportioned the same as a modern boa. A 10 foot boa weights 33 lbs.

(a) Use this information to estimate the wight of a 42 foot *Titanoboa*.

Let L = length
W = weight.
Then for some constant c

$W = cL^3$

But $W = 33$ when $L = 10$.

$33 = c(10)^3$

$c = \frac{33}{1000} = .033.$

so

$W = .033L^3$

when $L = 42$

$W = .033(42)^3$
$= 2444.9$ lbs

The weight is 2,444.9 lbs

(b) Estimate the length of a 1,000 lb *Titanoboa*.

solve

$1000 = .033L^3$

$L^3 = 1000/.033$

$L = (1000/.033)$^$(1/3)$

$= 31.17$

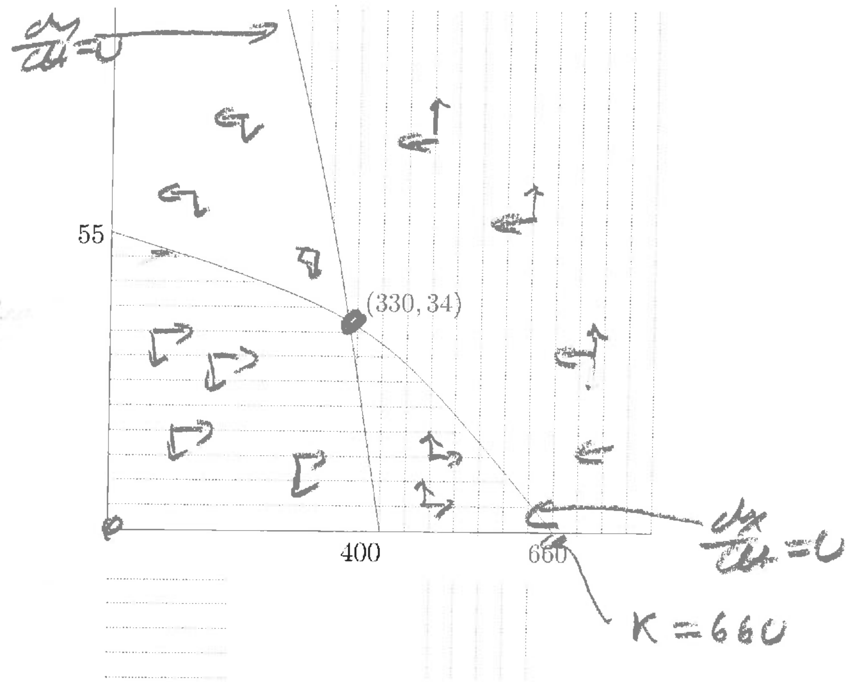
The length is 31.17 ft.

5.

Consider a system of rate equations relating the sizes of the populations of two species, the x-species and the y-species:

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y)$$

and assume the phase diagram looks like:



$f(x,y) > 0$ shaded like this.

$g(x,y) > 0$ shaded like this

(a) Label the curved line where $\frac{dx}{dt} = 0$

(b) Label the curved line where $\frac{dy}{dt} = 0$

(c) What are the equilibrium points? The points are (0, 0), (330, 34)

(d) Put in arrows which show which way a point is moving in each of the regions.

(e) If this is a predator-victim system which is the prey (x-species or y-species) and what is it carrying capacity? Explain your answer.

It is the x-species, as the y-species dies off if there is no x-species. K = 660