Mathematics 172

Quiz 36 Name: Key

You must show your work to get full credit.

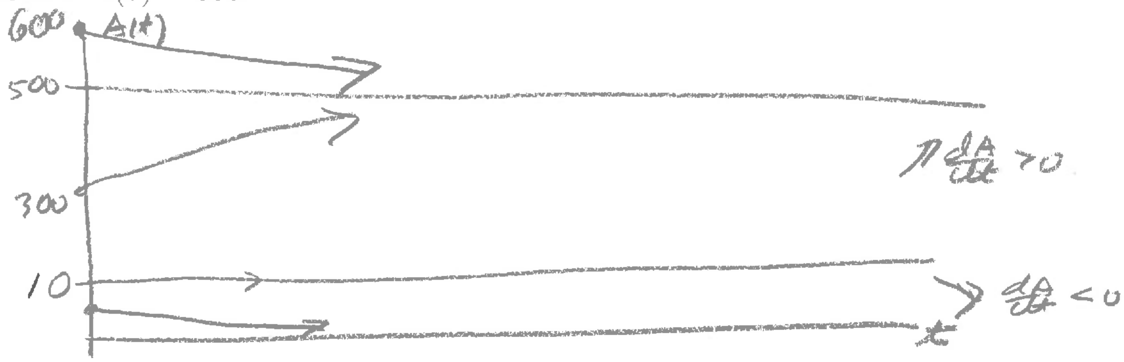
1. Water hyacinth is introduced into a pond. Let $N(t)$ be the number of pounds of it in the pond t weeks after in is introduced. Assume that A satisfies

$$\frac{dA}{dt} = .12A(A-10)(500-A)$$

(a) What are the equilibrium points of this rate equation?

The equilibrium points are: 0, 10, 500

(b) Sketch a graph showing the equilibrium solutions and also the solutions with $A(0) = 5$, $A(0) = 300$ and $A(0) = 600$.



(c) Which of the equilibrium points are stable and which are unstable:

The stable points are: 0, 500

The unstable points are: 10

(d) For the solution with $A(0) = 5$ estimate $A(85)$. $A(85) \approx$ 0

(e) For the solution with $A(0) = 300$ estimate $A(85)$. $A(85) \approx$ 500

2. Water fleas are breading in a bucket. To start assume this population grows logistically with an intrinsic growth rate of $r = .3$ (fleas/week)/flea and a carrying capacity of $K = 200$.

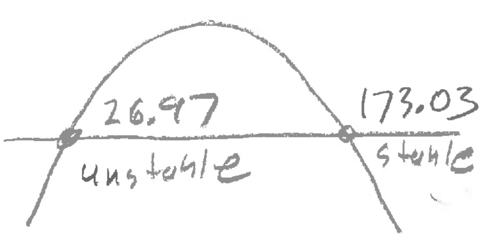
(a) Let $N(t)$ be the number of water fleas in the bucket in week t. What is the rate equation satisfied by $N(t)$?

The equation is: $\frac{dN}{dt} = .3N\left(1 - \frac{N}{200}\right)$

(b) Assume that after the population of water fleas has reached its carrying capacity that a single mosquito also is added to the bucket and it eats the water fleas at the constant rate of 7 fleas/week. What is the new rate equation satisfied by $N(t)$?

The equation is: $\frac{dN}{dt} = .3N\left(1 - \frac{N}{200}\right) - 7$

(c) What is the new stable population size of the water fleas?



The stable population size is: 173.03

$Y3 = .3X(1 - X/200) - 7$

Xmin = 0

Xmax = 200

ZoomFit

3. A population of fish is living in a polluted lake. Due to the pollution the intrinsic growth rate of the population is $r = -.08$ (fish/year)/fish. At what rate should the lake be stocked to have a stable population size of 5,000 fish?

Let N = number of fish in year t.

The stocking rate is: 400 fish/year

S = stocking rate

Then $\frac{dN}{dt} = -.08N + S$

$S = (.08)(5000) = 400$

we want $N = 5000$ to be a rest point so

$0 = -.08(5000) + S$

4. A new building for student housing is invaded by 5 cockroaches. Assume that with no constraints the roach population doubles every week. Then how long until there are a billion roaches?

The number of Roaches after t weeks is

Time to 1,000,000,000 roaches. 27.575 weeks

$N(t) = 5(2)^t$

so solve $5(2)^t = 10^9$

$2^t = 10^9$

$t\ln(2) = \ln(10^9/5)$

$t = \ln(10^9/5)/\ln(2)$

$= 27.575$

5. Assume that $P'(t) = .15P(t)$ and $P(0) = 42$.

(a) Give a formula for $P(t)$.

$P(t) =$ $42e^{.15t}$

$P(t) = P_0 e^{rt}$

$= 42e^{.15t}$

(b) What is the doubling time of $P(t)$?

Doubling time is 4.62

solve $42e^{.15t} = 2(42)$

$e^{.15t} = 2$

$.15t = \ln(2)$

$t = \ln(2)/.15 = 4.62$

6. Assume that 15 rabbits are released on a island that has no rabbits. Assume that this population grows exponentially and that a survey 3 years later finds there are 50 rabbits.

(a) Give a formula for the number, $N(t)$, of rabbits after t years.

$N(t) = N(0)\lambda^t = 15\lambda^t$

$N(3) = 15\lambda^3 = 50$

$\lambda^3 = \frac{50}{15}$

$\lambda = (50/15)$^$(1/3) = 1.494$

$N(t) =$ $5(1.499)^t$

(b) What is the per capita growth rate of the rabbits?

$r =$ $\lambda - 1 = .49$ rabbits/rabbit.

(c) Can this exponential growth hold indefinitely? Why?

No, it would be unbounded and overrun resources.

7. A population grows according to the discrete logistic equation

$$N_{t+1} = N_t + .3N_t\left(1 - \frac{N_t}{200}\right) \qquad \text{and} \qquad N_0 = 150.$$

(a) Find the following

$N_1 =$ 161.25 $\qquad N_2 =$ 170.62 $\qquad N_3 =$ 178.14

$Y1 = X + .3X(1 - X/200)$

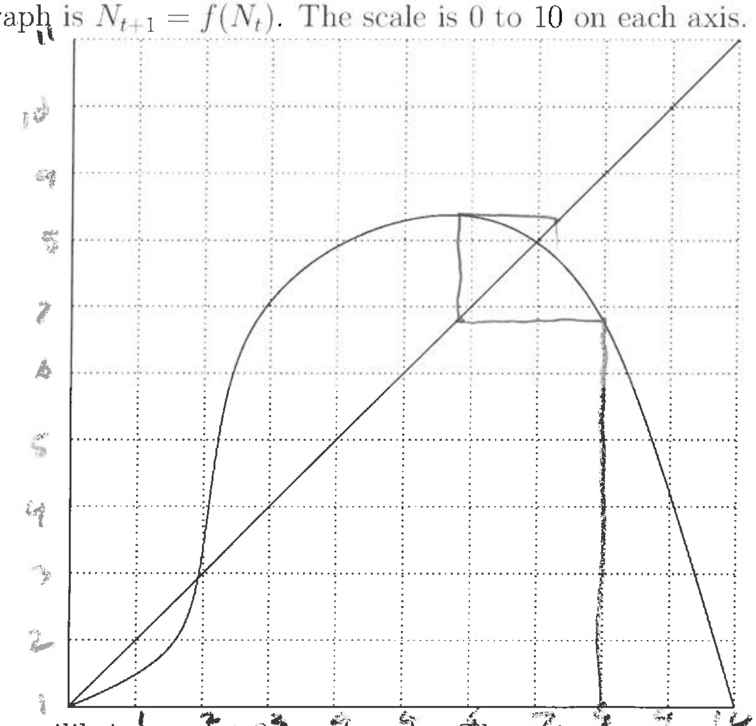
use 2nd calc value.

(b) What is the carrying capacity? $\qquad K =$ 200

(c) Estimate the following

$N_{50} =$ 200 $\qquad N_{51} =$ 200 $\qquad N_{123} =$ 200

8. The following graph is $N_{t+1} = f(N_t)$. The scale is 0 to 10 on each axis.



(a) What are the equilibrium points? $\qquad$ The points are: 0, 3, 7

(b) Which of these points are stable? $\qquad$ Stable points are: 0, 7

(c) If $N_0 = 8$ estimate the following

$N_1 \approx$ 5.8 $\qquad N_2 \approx$ 7.3 $\qquad N_{100} \approx$ 7