Mathematics 172 Test 1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 points) An hobbyist sets up an aquarium. At some point he buys a plant for the aquarium that has .15 grams of a fibrous algae on it. In the tank the algae grows with an intrinsic growth rate of $r = .3(grams/gram)/week$. Let $P(t)$ be the number of grams of the algae after t weeks.

(a) Give a formula for $P(t)$. $P(t) =$ $.15e^{.3t}$

In general $P(t) = P(0)e^{rt}$

In our case this is $P(t) = .15e^{.3t}$

(b) How long until there is 100 grams of the algae in the tank?

Time to 100 grams is 21.65 days (or round up to 22 days)

Solve

$$P(t) = .15e^{.3t} = 100$$

$$e^{.3t} = 100/.15$$

$$.3t = \ln(100/.15)$$

$$t = \ln(100/.15)/.3$$

$$= 21.67$$

2. (10 points) A reflecting pool in the lobby of a hotel is stocked with koi. Due to guests at the hotel throwing coins (which are poison to the fish) in the pool, the per capita grow rate of the koi population is $r = -.2$ (fish/fish)/year. The management of the hotel wants to keep a population of 50 koi in the pool. At what rate should they stock the pool?

Stocking rate is 10 fish/year

Let $N(t)$ = number of koi in year t, and S be the stocking rate.

The rate equation of N is

$$\frac{dN}{dt} = -.2N + S$$

We want $N = 50$ to be an equilibrium point so this gives

$$0 = -.2(50) + S$$

$$S = .2(50) = 10$$

3. (20 points) Yeast is growing in a large bucket of water. Let $W(t)$ be the weight in grams of the yeast in the bucket after t days. Assume that W satisfies the rate equation

$$\frac{dW}{dt} = .25W\left(1 - \frac{W}{50}\right)\left(\frac{W}{10} - 1\right).$$

(a) If $W(4) = 40$, what is $W'(4)$? $W'(4) =$ 6 grams/day

$W'(4) = .25(40)(1 - \frac{40}{50})(\frac{40}{10} - 1) = 6$

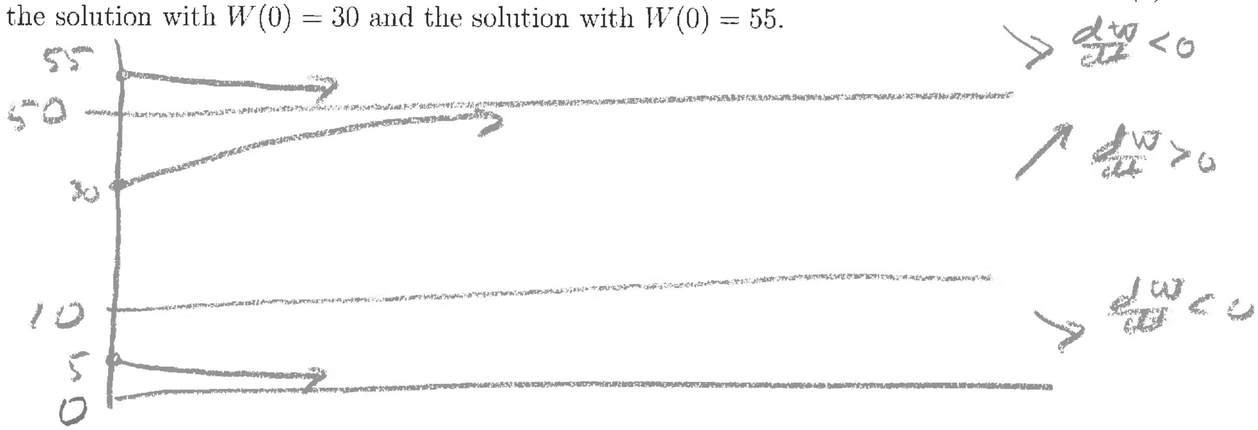
(b) What are the equilibrium points of this equation?

The equilibrium points are: 0, 10, 50

Solve

$.25W(1 - \frac{W}{50})(\frac{W}{10} - 1) = 0$ to get $W = 0, 10, 50$

(c) Draw a picture which shows the equilibrium solutions and also the solution with $W(0) = 5$, the solution with $W(0) = 30$ and the solution with $W(0) = 55$.



(d) Which of the equilibrium points are stable?

The stable points are: 0, 50

The equilibrium points with paths converging to them.

(e) If $W(0) = 5$, estimate $W(100)$. $W(100) \approx$ 0

Path going down to 0

(f) If $W(0) = 30$, estimate $W(93)$. $W(93) \approx$ 50

Path going up to 50

4. (15 points) A national park that has not had wolves in it for 42 years has a population of 15 wolves released. Assume that that population of wolves grows with a discrete logistic law with a per capita growth rate of $r = .15$ wolves/wolf and a carrying capacity of $K = 75$ wolves. Let N_t be the size of the wolf population in the park t years after they are released.

(a) Write down the discrete dynamical system satisfied by N_t.

The discrete logistic eqn is

$N_{t+1} = N_t + rN_t(1 - \frac{N_t}{K})$

Here $r = .15$, $K = 75$

$N_{t+1} =$ $N_t + .15N_t(1 - \frac{N_t}{75})$

(b) What are N_1 and N_2?

$N_1 =$ 16.8

$N_2 =$ 18.76

$N_1 = 15 + .15(15)(1 - \frac{15}{75}) = 16.8$

$N_2 = 16.8 + .15(16.8)(1 - \frac{16.8}{75})$

$= 18.76$

(c) Estimate N_{50}. Here $r = .15$ so $0 < r < 2$. Thus $K = 75$ is a stable equilibrium point. So points converge into $K = 75$

$N_{50} \approx$ 75

5. (10 points) A population of duckweed is growing logistically in a pond with an intrinsic growth rate of $r = 1.8$ (lbs/lb)/week and a carrying capacity of $K = 500$ pounds. The owner of the pond wishes to get rid of the duckweed. What is the least rate she can harvest it so that it eventually is eradicated. Write a sentence or two and include a picture explaining how you got the answer.

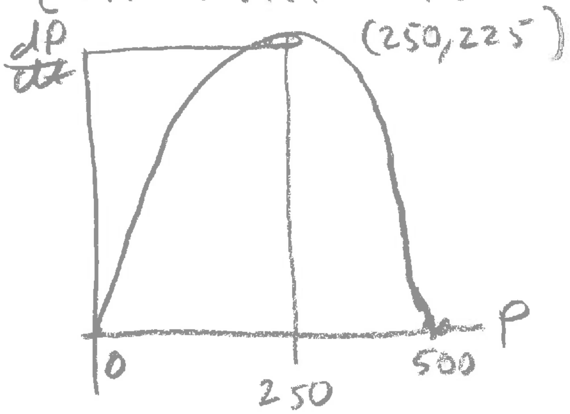
Harvesting rate is (include units) 225 lbs / week.

Let $P(t)$ = pounds of duckweed in week t. Then

$\frac{dP}{dt} = 1.8P(1 - \frac{P}{500})$

Plot $\frac{dP}{dt}$ as a function of P

$(Y = 1.8X(1 - X/500))$

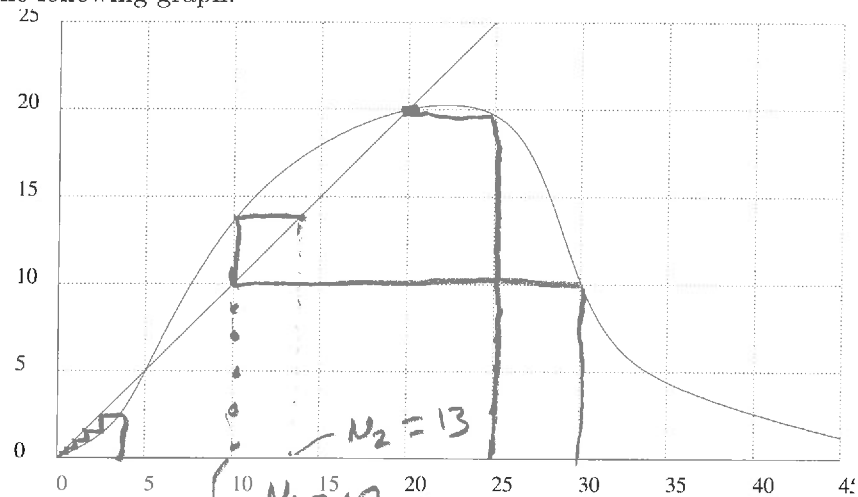


The maximum of $\frac{dP}{dt}$ is 225 lbs/week. So if she harvest 225 lbs/week or more the rate will be negative and the duckweed will die out.

6. (20 points) My backyard has a pond that supports a population of frogs. Let N_t be the number of frogs t years after I first started counting them. Assume that

$$N_{t+1} = f(N_t)$$

where f has the following graph:



(a) What are the equilibrium points of this system?

This is where the graph crosses $y=x$

Equilibrium points are $0, 5, 20$

(b) Which of the equilibrium points are stable $0, 20$

where $|\text{slope}| < 1$

(c) Which of the equilibrium points are unstable 5

where $|\text{slope}| > 1$

(d) If we start with $N_0 = 30$ frogs, then estimate N_1 and N_2.

$N_1 \approx$ 10

See cobweb

$N_2 \approx$ 13

(13.5 or 14 also ok)

(e) If we start with 3 frogs (that is $N_0 = 3$) estimate N_{50}. $N_{50} \approx$ 0

Starting at 3, it cobwebs down to 0.

(f) If $N_0 = 25$ estimate N_{60}. $N_{60} \approx$ 20

Starting at 25, it cobwebs to 20

7. (20 points) Some squirrels are living on a small island. Let P_t be the size of the squirrel population in year t. Assume that if the population size in year t is P_t, the population size the next year is

$$P_{t+1} = P_t e^{.2(1-P_t/200)}$$

(a) If $P_0 = 180$ compute P_1 and P_2 accurate to 2 decimal places.

To make the computation easier I entered

$\backslash Y1 = Xe^{\wedge}(.2(1-X/200))$

Xmin = 0, Xmax = 220

2nd calc 1: value $X = 180$ $P_1 = 183.64$

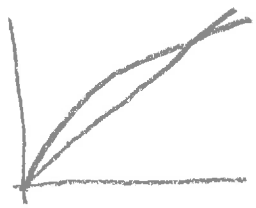
2nd calc 1: value $X = 183.64$ $P_2 = 186.67$

$P_1 =$ 183.64

$P_2 =$ 186.67

(b) Use your calculator to find the equilibrium points.

with \Y1 as above use \Y2 = X graph



The equilibrium points are: 0, 200

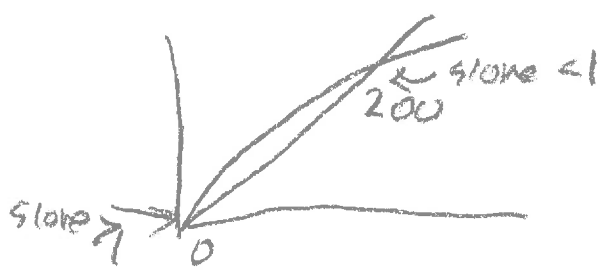
The stable equilibrium points are: 200

The unstable equilibrium points are: 0

One eqm pt is 0
Use 2nd calc 5: intersect to find that the other one is 200

(c) Give a sentence or two explaining how you determined how the points were stable or unstable. (This explanation may involve computing some derivatives (i.e dy/dx) on the calculator.)

One way is to just look at the graph and see that at $P_* = 0$ slope > 1 so unstable and at $P_* = 200$ slope < 1 so stable.



or At $P_* = 0$ $\frac{dy}{dx} = 1.22 > 0$ unstable

At $P_* = 200$ $\frac{dy}{dx} = .8 < 1$ stable