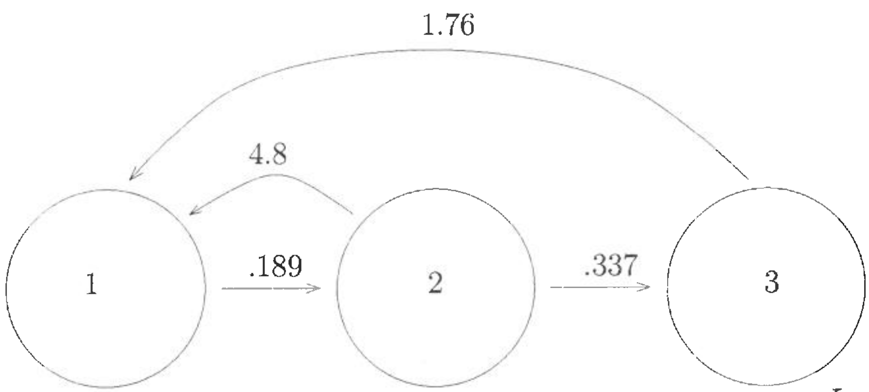
Mathematics 172 Test 2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (20 points) Some species killifish breed just once a year and live to be a most two years old. We look at three life stages. Stage 1: hatchling, Stage 2: one year old, and Stage 2: two years old. The life history of a population of killifish living in the ponds, puddles, and streams of a small island is summarized by the loop diagram:



(a) What is the Leslie matrix?

$$L = \begin{bmatrix} 0 & 4.8 & 1.76 \\ .189 & 0 & 0 \\ 0 & .337 & 0 \end{bmatrix}$$

(b) What does the number 4.8 mean?
Average number of offspring to a stage 2 individual that live to stage 1.

(c) What does the number .337 mean? The proportion of stage 2 individuals that live to stage 3.

(d) What proportion of hatchlings live to be two years old? Proportion is .06393

$(.189) \times (.337) = .06393$

(e) If this year there are 502 hatchlings, 81 one year olds, and 26 two year olds, then after 20 years how many are in each stage and what proportion are in each stage?
Number in each stage:

Stage 1 560.137 Stage 2 103.602 Stage 3 35.098

total = 698.84

Proportion in each stage:

Stage 1 .802 $= \frac{560.137}{698.48}$ Stage 2 .148 $= \frac{103.603}{698.48}$ Stage 3 .050 $= \frac{35.098}{648.46}$

2. (20 points) A population of killifish on a different island has its life history summarized by the Leslie matrix:

$$L = \begin{bmatrix} 0 & 4.8 & 1.2 \\ 0.204 & 0 & 0 \\ 0 & 0.255 & 0 \end{bmatrix}.$$

This matrix has an eigenvalue of $\lambda = 1.02$ and an eigenvector of

$$\begin{bmatrix} 40 \\ 8 \\ 2 \end{bmatrix}.$$

(a) What is the per capita growth rate? $r =$ $\lambda - 1 = .02$

(b) What is the stable age distribution?

An eigenvector is at the stable age distribution, total $= 40+8+2 = 50$

Proportion in Stage 1: $\frac{40}{50} = .8$

Proportion in Stage 2: $\frac{8}{50} = .16$

Proportion in Stage 3: $\frac{2}{50} = .02$

(c) If the population has settled down to its stable age distribution and in some year there are 200 hatchlings, then how many hatchlings are there the next year?

The number of hatchlings is 204.

The percapita growth rate at the stable distribution is .02 so

$200 + (.02)(200) = 204$

3. (20 points) For the initial value problem

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{20}\right) \qquad P(0) = 15$$

(a) Do two steps of length of Euler's method of length $h = .25$ to estimate $P(.5)$.

$P(.5) \approx$ 15.279

step 1: $P'(0) = .15(15)(1 - \frac{15}{20}) = .5625$

$P(.25) \approx P(0) + P'(0)(.25) = 15 + (.5625)(.25) = 15.141$

step 2: $P'(.25) \approx .15(15.141)(1 - \frac{15.141}{20}) = .5518$

$P(.5) \approx P(.25) + P'(.25)(.25)$

$= 15.141 + (.5518)(.25) = 15.279$

(b) Estimate $P(79)$ $\approx$ carrying capacity $= 20$ $P(79) \approx$ 20

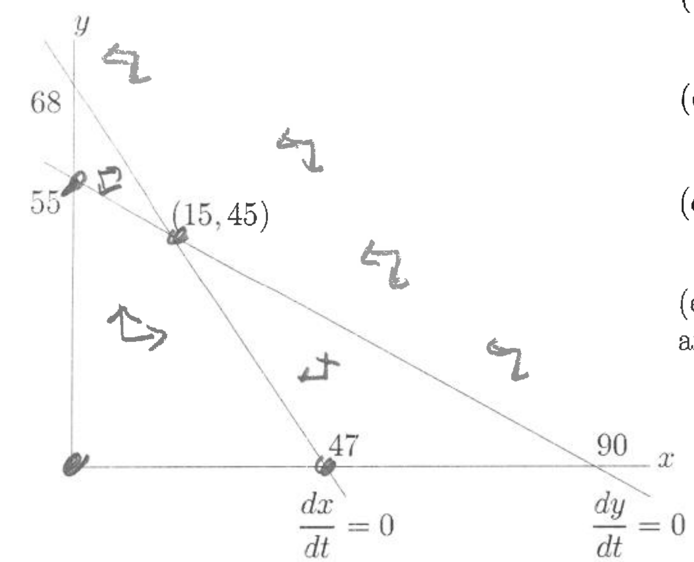
4. (20 points) The following are phase diagrams for the equations

$$\frac{dx}{dt} = r_1 \left(\frac{K_1 - x - \alpha y}{K_1} \right)$$
$$\frac{dy}{dt} = r_2 \left(\frac{K_2 - \beta x - y}{K_2} \right)$$

of competing species.

(a) Fill in arrows showing in what direction that points are moving in each region.

$(K_1, 0) =$ x intercept of $x + \alpha y = K_1$



(b) What is K_1?

$K_1 =$ 47

(c) What are the rest points?

Rest points are: (0,0), (47,0), (0,55), (15,45)

(d) What are the stable rest points?

Stable points are: (15,45)

(e) If $x(0) = 15$ and $y(0) = 50$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 15

$y(100) \approx$ 45

converges to the stable point

(f) Which of the following describes the long term behavior of this system (circle one).

Completive coexistence ***Completive exclusion***

x-species dominates ***y-species dominates.***

(g) If there is no x-species present, what is the stable y-population size?

55 = y-carry capacity

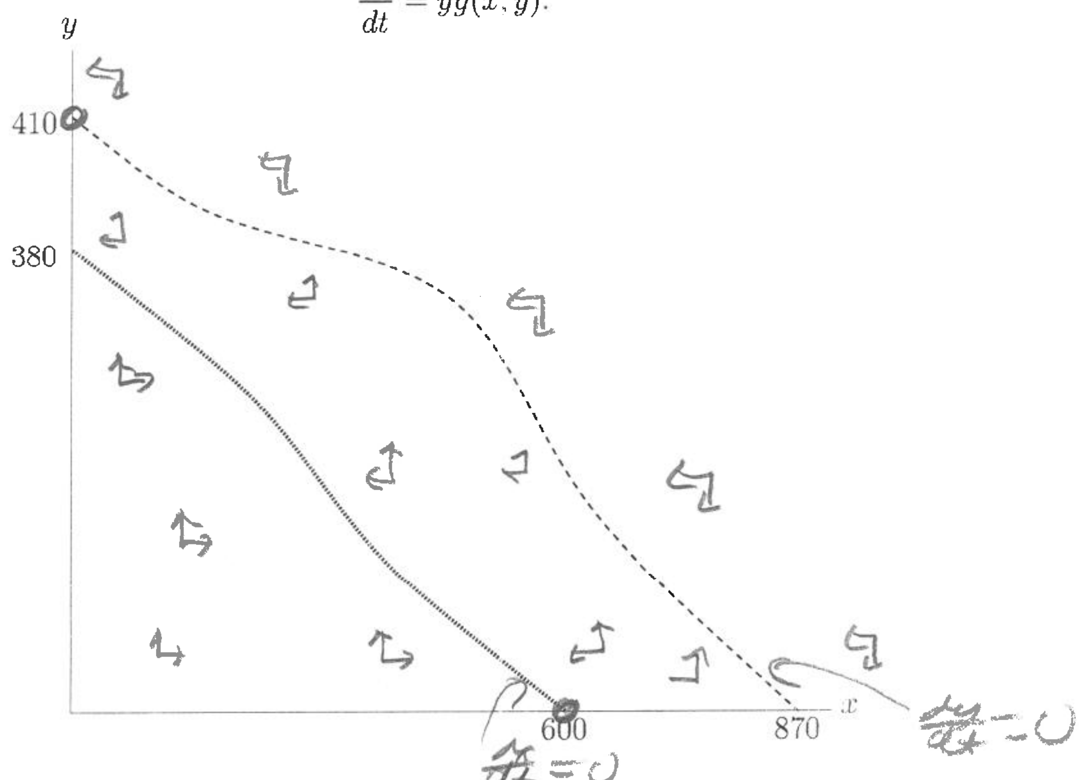
Stable size is 55

(h) Assume that to begin with there are no y-species present and there is a stable x population of size 47. Then 5 of the y-species is introduced. What happens in the long run?

The y-species invades, y increases to 45, and x decreases to 15.

5. (20 points) The figure below is the phase space for the system

$$\frac{dx}{dt} = xf(x, y)$$
$$\frac{dy}{dt} = yg(x, y).$$



-------------- $g(x, y) = 0$ on this curve and is positive below it.

.... $f(x, y) = 0$ on this curve and is positive below it.

(a) Fill in the arrows showing what direction points are moving in each region.

(b) What are the rest points? Rest points are: (0,0), (600,0), (0,410)

(c) What are the stable rest points Stable points are: (0,410)

(d) If there is no x-species present, what is the carrying capacity for the y-species?

Capacity is 410

(e) If there is no y-species present, what is the carrying capacity for the x-species?

Capacity is 600

(f) If at first there is no y-species present, and the x-species is at its carrying capacity and 30 of the y-species are introduced to the region, then what happens in the long run?

The y-species invades increases to 410 and forces the x-species to extinction.