Mathematics 172 Test 3 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (25 points) For the predator-victim system

$$\hat{P} = \frac{.1}{.002} = 50$$

$$\frac{dV}{dt} = .1V - .002VP = V(.1 - .002P)$$

$$\frac{dP}{dt} = -.5P + .0005VP = P(1 - .0005V)$$

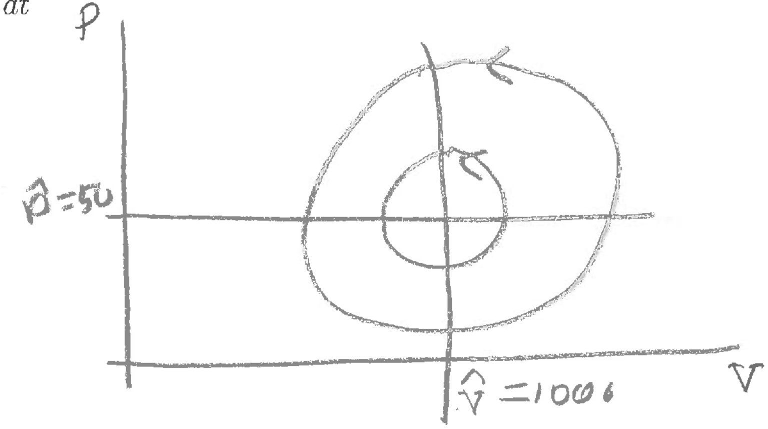
$$\hat{V} = \frac{.5}{.0005} = 1000$$

(a) If there are no predators, what is the equation for V?

(b) Is the equation for V with no predators realistic for long term behavior? Why?

If there are no predators, i.e. $P = 0$, then $\frac{dV}{dt} = .1V - 0 = .1V$. This has solution $V(t) = V(0)e^{.1t}$ which becomes unbounded and so would out grow all resources.

(c) Draw the phase space space (V on the x-axis and P on the y-axis) and label the lines where $\frac{dV}{dt} = 0$ and $\frac{dP}{dt} = 0$ and draw in a few of the cycles.



(d) What are the are the average number of victims and predators?

$\hat{V} =$ 1000 $\hat{P} =$ 50

(e) What happens to the average number of victims and predators if the death rate of the predator is doubled?

New $\hat{V} =$ 2000 New $\hat{P} =$ 50

This leaves the $\frac{dV}{dt}$ equation unchanged and so $\hat{P} = 50$ still holds. The second equation becomes $\frac{dP}{dt} = -.1P + .0005VP = P(-.1 + .0005V)$ so new $\hat{V} = \frac{.1}{.0005} = 2000$

2. (20 points) For the predator-victim system

$$\frac{dV}{dt} = .15V\left(1 - \frac{V}{600}\right) - .0075VP$$

$$\frac{dP}{dt} = -.6P + .002VP$$

(a) What is the carrying capacity of the victim population if there are no predators?

$K =$ 600

(b) Draw the phase plane of the system showing the rest points and with arrows showing the direction of motion.

Rest points are (0,0), (600,0), (300,10)

$\frac{dV}{dt} = V\left(.15\left(1-\frac{V}{600}\right) - .0075P\right)$

so $\frac{dV}{dt} = 0$ on $V=0$ and $.15\left(1-\frac{V}{600}\right) - .0075P = 0$

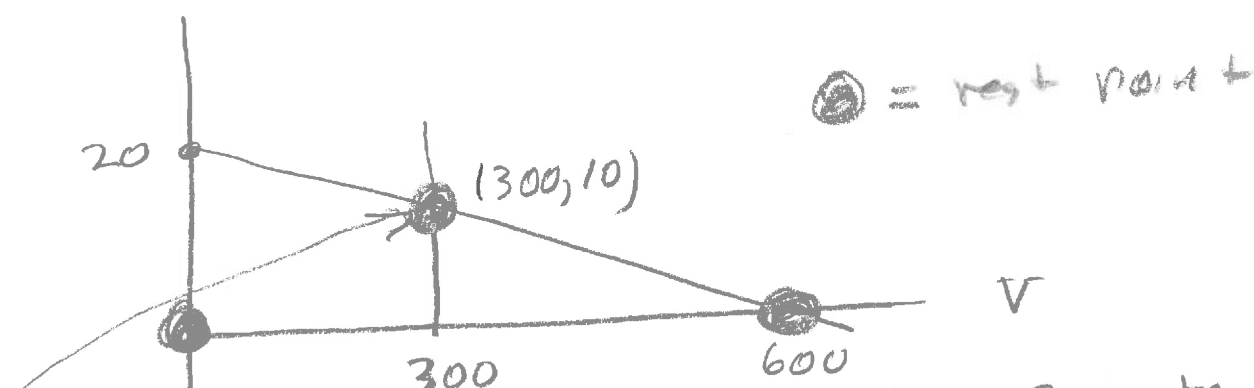
For the second the V intercept is when

$.15\left(1-\frac{V}{600}\right)=0$, i.e. $V=600$.

The P intercept is when $.15 - .0075P = 0$,

i.e. $P = \frac{.15}{.0075} = 20$.

$\frac{dP}{dt} = P(-.6 + .002V)$. So $\hat{V} = \frac{.6}{.002} = 300$



use $V=300$ in $.15\left(1-\frac{V}{600}\right) - .005P = 0$ to get $.15\left(1-\frac{300}{600}\right) - .005P = 0$ $P = \frac{(.15)(.5)}{.005} = 10$

3. (25 points) Consider the following version of an SIR model where recovered lose their immunity after a period of time:

$$S' = -.002SI + .01R$$
$$I' = .002SI - .1I$$
$$R' = .1I - .01R$$

(a) What is the average duration of an infections?

$\text{Duration} = \frac{1}{\text{infection rate}} = \frac{1}{.1} = 10$ Duration is 10 days

(b) What is the average length of time an individual keeps their immunity?

Duration is 100

$\text{immunity duration} = \frac{1}{\text{rate of loss of immunity}} = \frac{1}{.01} = 100$

(c) If $S(9) = 90$, $I(9) = 7$ and $R(9) = 3$ do one step of length 1 in Euler's method to estimate $I(10)$.

$I(10) \approx$ 7.56

$I'(9) = .002S(9)I(9) - .1I(9)$
$= .002(90)(7) - .1(7)$
$= .56$

so

$I(10) \approx I(9) + I'(9)(1)$
$= 7 + .56$
$= 7.56$

(d) There is a cut off, c, in the size of S such that if S becomes less that c, then I starts decreasing. What is the value of c?

$c =$ 50

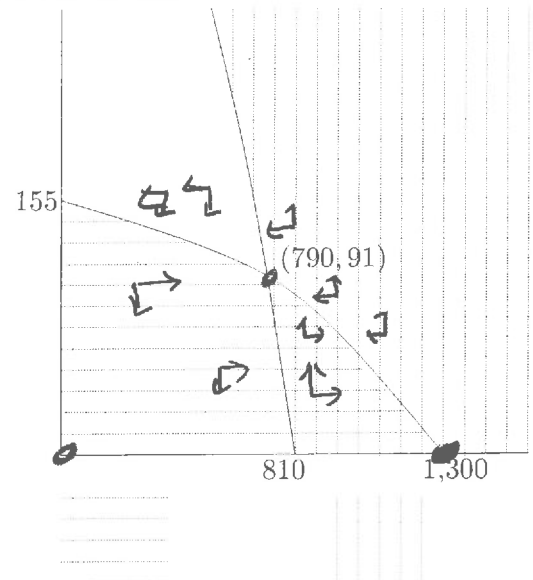
$I' = I(.002S - .1)$

changes sign when $S = \frac{.1}{.002} = 50$

4. (20 points) Consider a system of rate equations relating the sizes of the populations of two species, the x-species and the y-species:

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y)$$

and assume the phase diagram looks like:



$f(x,y) > 0$ shaded like this.

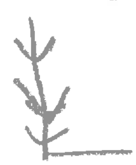
$g(x,y) > 0$ shaded like this

(a) Put in arrows which show which way a point is moving in each of the regions.

(b) What are the rest points? The points are (0,0), (790,91), (0,1300)

(c) If there is no x-species what happens to the y-species.

If $x=0$ we are on the y-axis so y decreases to zero



(d) If there is no y-species what happens to the x-species?

If $y=0$ we are on the x-axis

The population levels off at 1300 = carrying capacity

5. (10 points) The amount of skin on a python is proportional to its surface. Assume that a 2 meter long python has a surface area of .6 m^2.

(a) How much skin does a 5 meter long python have?

Amount of skin. 3.75 m^2

$A = cL^2$ (area proportional to square of length)

when $L = 2$, $A = .6$ so

$.6 = c\,2^2$

$c = \frac{.6}{2^2} = .15$

$\underline{\underline{A = .15L^2}}$

when $L = 5$

$A = .15(5)^2 = 3.75$

(b) Someone needs 1.2 m^2 of python skin to make a pair of boots. How long a python does she need to catch to make sure she has enough python skin.

The minimum length is 2.828 m.

$A = .15L^2$

we need at least $A = 1.2\,m^2$

so solve $.15L^2 = 1.2$

$L^2 = \frac{1.2}{.15}$

$L = \sqrt{\frac{1.2}{.15}} = 2.828$