Mathematics 551 Homework, March 28, 2020

Here is reading and homework to look at. First read Section 2.2 (Pages 44–53) in Shifrin's Differential Geometry: A First Course in Curves and Surfaces. We have been over this material, but it is something that took me several passes to learn and so I hope that seeing it from points of view other than mine will be helpful.

Problem 1. Look at Example 6 on Page 49 of Shifrin and as a variant on it consider the surface parametrized by

$$\mathbf{x}(u,v) = \left(u, v, \frac{au^2 + bv^2}{2}\right)$$

where a and b are constants. Following Shifrin's calculations and notation compute I_P, II_P, S_P, k_1, k_2, the mean curvature H, and the Gauss Curvature K. $\square$

Problem 2. Using the notation

$$\begin{aligned} E_1(v) &= (\cos(v), \sin(v), 0) \\ E_2(v) &= E_1'(v) = (-\sin(v), \cos(v), 0) \\ E_3(v) &= (0, 0, 1) \end{aligned}$$

define

$$\mathbf{x}(u,v) = uE_1(v) + vE_3 = (u\cos(v), u\sin(v), v).$$

Compute I_P, II_P, S_P, k_1, k_2, the mean curvature H, and the Gauss Curvature K for this surface. $\square$

Problem 3. Let $\mathbf{c}\colon I(a,b)\colon \mathbb{R}^3$ be a unit speed curve and use the usual notation for the unit tangent, unit normal, the bi-normal etc. Assume that $\kappa \neq 0$ on this curve. Define

$$\mathbf{x}(u,v) = \mathbf{c}(u) + v\mathbf{t}$$

for $u \in (a,b)$ and $v > 0$.

(a) Show that the unit normal to this surface is, $\mathbf{b}$, the bi-normal to the curve.
(b) Show that the Gauss curvature of the surface is identically zero. $\square$